Budapest University of Technology and Economics

Department of Mechanics, Materials and Structures

English courses

Reinforced Concrete Structures

Code: BMEEPSTK601

Lecture no. 9:

TWO-WAY SLABS

Content:

1. Definition, internal force components
2. Methods of analysis of two-way slabs
3. The method of Marcus
4. The effect of fixing the corners of the slab against lifting
5. The yield line theory
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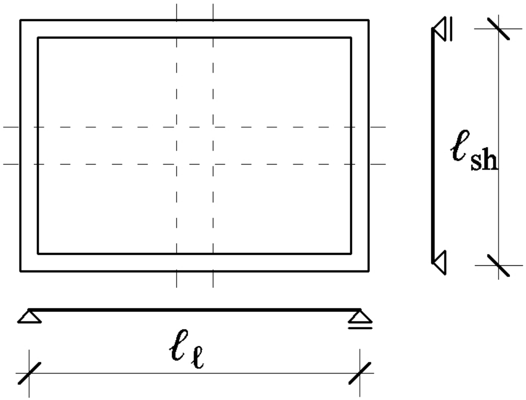
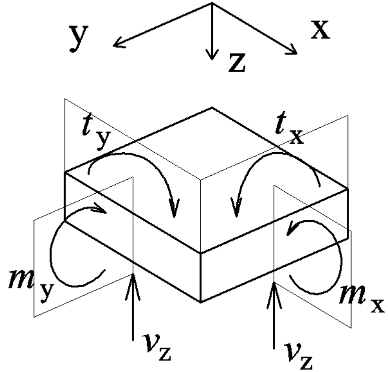
1. Definition, internal force components





specific force components in unit-length sections of two-way slabs

If $\ell_{sh} \geq 0{,}5\ell_{\ell}$, the interaction of perpendicular strips of the slab through torsion (t_x and t_y (kNm/m)) is taken into account by determining the internal force distribution: the slab is regarded two-way slab

2. Methods of analysis of two-way slabs

The partial differential equation of slabs:

$$\frac{\partial^4 w}{\partial x^4} + \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial x^2 \partial y^2} = -\frac{p}{EI}$$

w: deflection   *p*=*p*(x,y) load

Analythical solutions

-exact: were only elaborated for very simple cases (for example: uniformly distributed load, rectangular slab, simply supported along the perimeter, EI= constant)

-approximate analythical solutions by use of Fourier functions for limited no. of cases

Numerical solutions can be computerized:

-method of finite differences, finite element method (FEM)

Results of both analythical and numerical solutions are available in *tabulated form*.

Two *manual approximate methods* will be shown below.

3. The method of Marcus

For rectangular slab, simply supported along the perimeter, loaded with uniformly distributed load.
The load is distributed between series of two perpendicular strips by considering the condition, that at the intersection of two perpendicular strips the deflection is the same (effect of torsion is neglected):

$$w_\ell = \frac{5}{384}\frac{p_\ell \ell_\ell^4}{EI} = w_{sh} = \frac{5}{384}\frac{p_{sh}\ell_{sh}^4}{EI} \quad \rightarrow \frac{p_\ell}{p_{sh}} = \left(\frac{\ell_{sh}}{\ell_\ell}\right)^4 \qquad (1)$$

$$p = p_{sh} + p_\ell \qquad (2)$$

By introducing the parameters $\alpha_{sh} = \frac{p_{sh}}{p}$ and $\alpha_\ell = \frac{p_\ell}{p}$

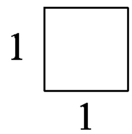
$$\alpha_{sh} = \frac{1}{1+\left(\frac{\ell_{sh}}{\ell_{\ell}}\right)^4} \qquad \alpha_{\ell} = 1 - \alpha_{sh}$$

The load intensities can then be determined:

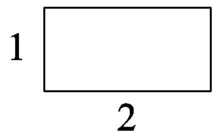
$$p_{sh} = \alpha_{sh} p \quad \text{and} \quad p_{\ell} = \alpha_{\ell} p$$

For the two extreme cases, this yields in:

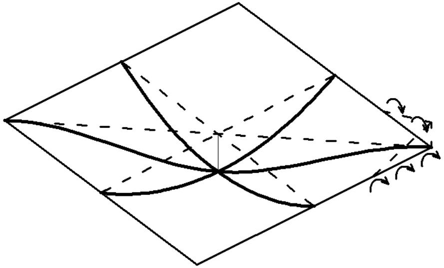
$$\alpha_{sh} = \alpha_{\ell} = 0,5$$



$$\alpha_{sh} \approx 0,9 \quad \alpha_{\ell} \approx 0,1$$

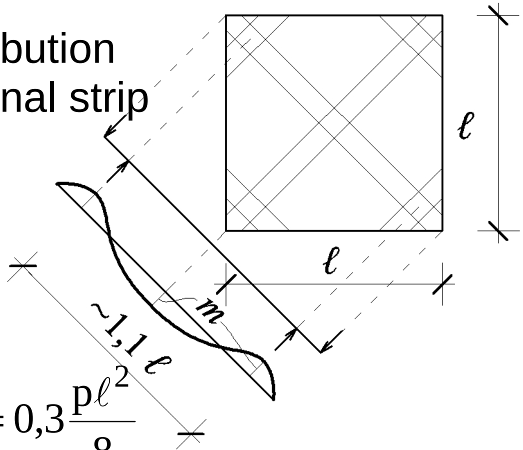


4. The effect of fixing the corners of the slab against lifting

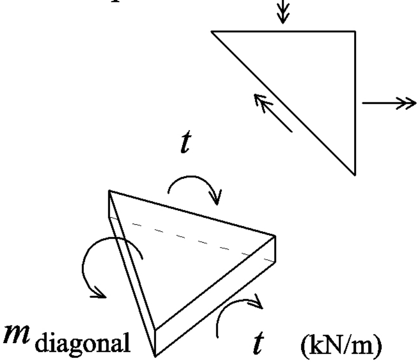


the corners of the slab lift, if they are not loaded by vertical forces of constructions above, resulting in torsional moments

Moment distribution along a diagonal strip

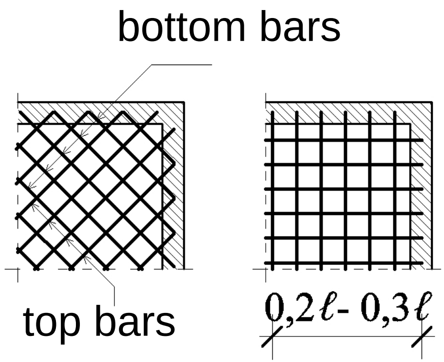


Moments' equilibrium at corners



$$m = \frac{\frac{p}{2}(1{,}1\ell)^2}{2\cdot 8} = 0{,}3\frac{p\ell^2}{8}$$

Top reinforcement designed for negative moments



in practice:
top mesh reinforcement parallel to sup-ports is used with the same intensity as that, designed for positive moments

5. The yield line theory

Yield lines: straight lines along primary cracks

Along yield lines: $m=m_{max}$, $v=0$

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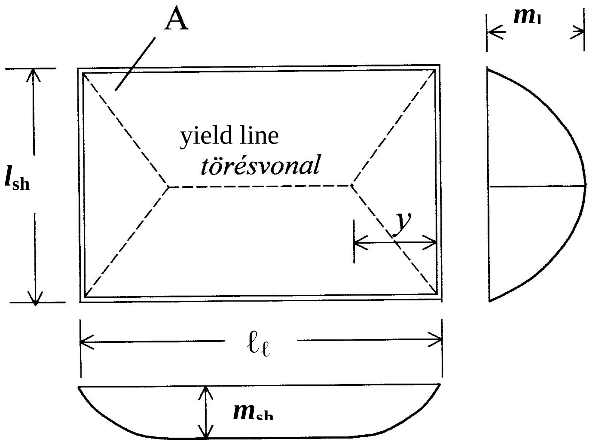


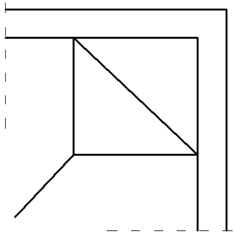
$y = \eta \ell_\ell$ where η can be freely adopted between 0,1 and 0,5 .

For example:

$$\eta_{opt} = \frac{1}{2}\left(\frac{l_{sh}}{l_l}\right)^2$$

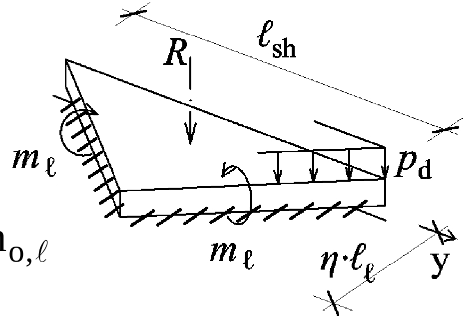
(results min. quantity of reinforcement)

More exact model at corners:



Equilibrium of triangular and trapezoidal panels of the slab

Consider one of the *triangular panels* and the equilibrium of moments with respect to axis *y* :

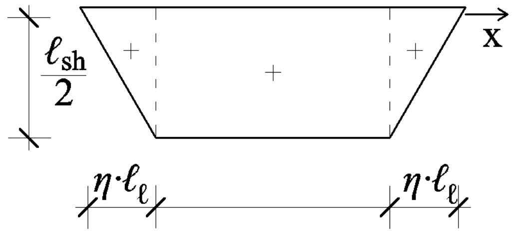


$\Sigma M_y=0$:
$$\frac{p\ell_{sh}y}{2}\frac{y}{3} = m_\ell \ell_{sh}$$

$$m_\ell = \frac{py^2}{6} = \eta^2 \frac{\ell_\ell^2 p}{6} = \frac{8}{6}\eta^2 \frac{p\ell_\ell^2}{8} = \frac{4}{3}\eta^2 \frac{p\ell_\ell^2}{8} = \alpha_\ell m_{o,\ell}$$

where $\alpha_\ell = \frac{4}{3}\eta^2$ and $m_{o,\ell} = \frac{p\ell_\ell^2}{8}$

Consider now one of the *trapezoidal panels* and the equilibrium of moments with respect to axis *x* :



$\Sigma M_x=0$:

$$2\cdot\frac{\eta\ell_\ell\ell_{sh}}{2}p\frac{\ell_{sh}}{2\cdot3}+(\ell_\ell-2\eta\ell_\ell)\frac{\ell_{sh}}{2}p\frac{\ell_{sh}}{2\cdot2}=m_{sh}\ell_\ell$$

Expressing m_{sh}, we get:

$$m_{sh}=(1-\frac{4}{3}\eta)\frac{p\ell_{sh}^2}{8}=\alpha_{sh}m_{o,sh}$$

where: $\alpha_{sh}=1-\frac{4}{3}\eta$ and $m_{o,sh}=\frac{p\ell_{sh}^2}{8}$

The load $p=p_{Ed}$ will be substituted in direction of the shorter and longer span respectively with modified values corresponding to the directions:

$$p_{sh}=\left(1-\frac{4}{3}\eta\right)p \qquad p_\ell=\frac{4}{3}\eta^2p \qquad (p_{sh}+p_l\le p)$$

6. Design conditions set up for the parameter *η* to determine the yield line pattern

Beside $\eta = \eta_{opt} = \frac{1}{2}\left(\frac{\ell_{sh}}{\ell_{\ell}}\right)^2$ some further conditions that can be set up to determine the value of the parameter η :

-let the steel necessary in direction of the longer span be equal to the area of the distribution steel of that necessary in direction of the shorter span: $a_{s,\ell}=0{,}2a_{s,sh}$ (or approximately: $m_{\ell}=0{,}2m_{sh}$)

-let the reinforcement be same in the two perpendicular directions:

$$a_{s,\ell}=a_{s,sh} \text{ (or approximately: } m_{\ell}=m_{sh})$$

-let the triangle of the yield line pattern be rectangular:

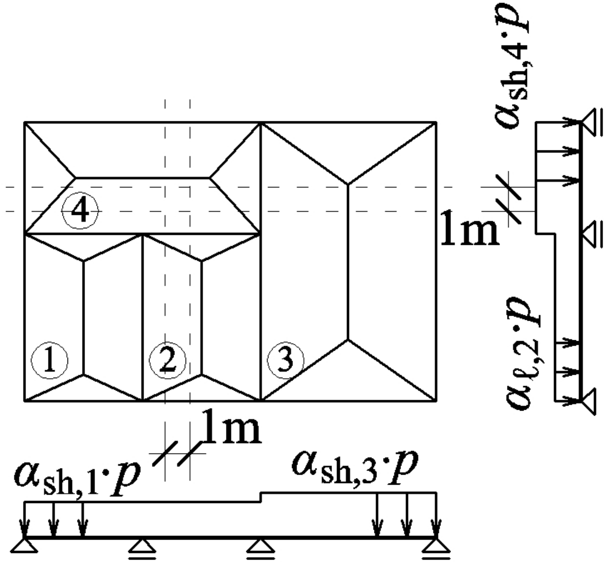
$$y = \eta\ell_{\ell} = 0{,}5\ell_{sh} \text{ that is: } \eta = 0{,}5\ell_{sh} / \ell_{\ell}$$

Limits of y determine limites of η, which should be checked:

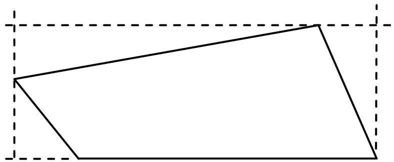
$$\frac{\ell_{\ell}}{10} \le y \le \frac{\ell_{\ell}}{2} \text{ that is: } 0{,}1 \le \eta \le 0{,}5$$

7. Application of the yield-line theory for more komplex situations:

-a system of continuous two-way slabs



-irregular ground plan forms can be completed to rectangle and handled like that

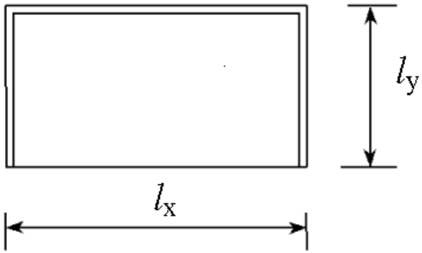


8. **Application of the yield line theory for different support condtions of rectangualr slabs and different ground plan forms**

-rectangular slab panel simply supported along three sides and free along the forth side:

the maximum moment:

$$m = \alpha \cdot p_{Ed} l_x^2 / 8,$$

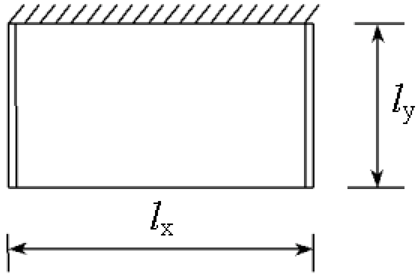


$$\alpha = \frac{2}{1{,}33 + \frac{l_x}{l_y}}$$

-rectangular slab with two free neighbouring edges and two restrained edges:

the maximum moment:

$$m = \alpha \cdot p_{Ed} l_x^2 / 8$$

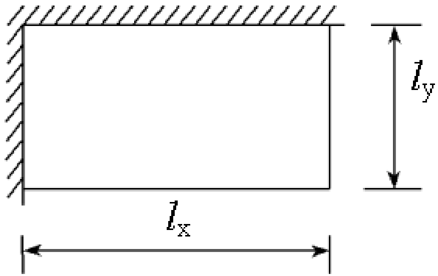


$$\alpha = \frac{2}{1{,}33 + \frac{l_x}{l_y} + 0{,}33\left(\frac{l_x}{l_y}\right)^2}$$

-rectangular slab with two free neighbouring edges and two restrained edges

Maximum moment:

$$m = \alpha \cdot p_{Ed} l_y^2 / 2$$

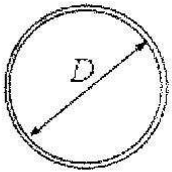


$$\alpha = 1 - 0{,}5\left(\frac{l_y}{l_x}\right)^2 \geq 0{,}5$$

Circular and other ground plan forms that can be characterized with inscribed circle, simply supported along the perimeter

$$m = \alpha \frac{p_{Ed} D^2}{8}$$

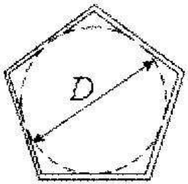
a) circle



$\alpha = 0{,}42$

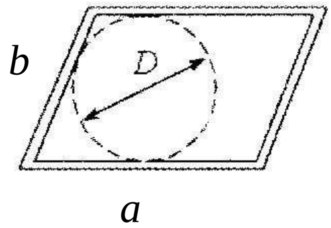
no top mesh needed here

b) polygon

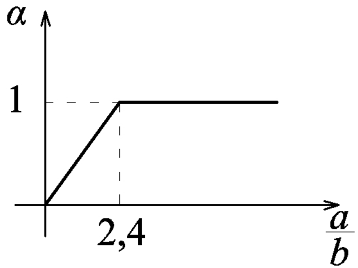


$\alpha = 0{,}42$

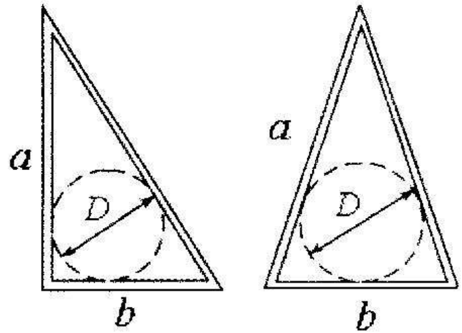
c) parallelogram



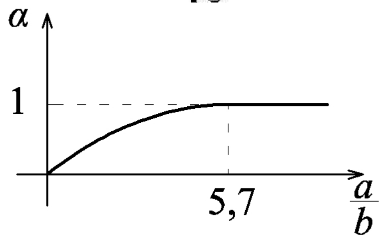
$\alpha = 0{,}42 \frac{a}{b} \leq 1$



d) triangle



$\alpha = 0{,}42 \sqrt{\frac{a}{b}} \leq 1$



9. Numerical example

Rectangular slab simply supported along the perimeter with optimal sidelength rate

Problemt

By what sidelength rate will the relationship – $m_{sh} = 5\, m_{\ell}$ be true just at $\eta = \eta_{opt}$? Lifting of the corners is impeded.

Solution

Let be: $$\frac{\ell_{sh}}{\ell_{\ell}} = \gamma$$

$$\eta_{opt} = \frac{1}{2}\gamma^2 \qquad (1)$$

$$(1-\frac{4}{3}\eta_{opt})\frac{p\ell_{sh}^2}{8} = 5.\frac{4}{3}\eta_{opt}^2\frac{p\ell_{\ell}^2}{8} \qquad (2)$$

From (2) with the substitution $\frac{\ell_{sh}}{\ell_{\ell}} = \gamma$ we get:

$$(1-\frac{4}{3}\eta_{opt})\gamma^2 = \frac{20}{3}\eta_{opt}^2$$

By expressing γ^2 and substituting it in (1):

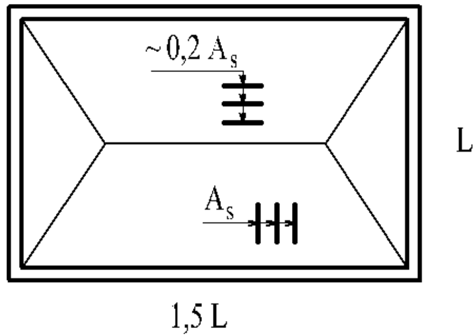
$$\eta_{opt}(1-\frac{4}{3}\eta_{opt}) = \frac{1}{2}\cdot\frac{20}{3}\eta_{opt}^2$$

We get the solution

$$\eta_{opt} = 0{,}214 = \frac{1}{2}(\frac{\ell_r}{\ell_h})^2 = \frac{1}{2}\gamma^2$$

$$\gamma = \sqrt{2 \cdot 0{,}214} = 0{,}654$$

That means: in case of ℓ_{sh} = 0,654ℓ_{ℓ}, or ℓ_{ℓ} = 1,528ℓ_{sh} that is at about by the sidelength rate 1:1,5 will the rate of moments in the two directions be equal to 1:5 just by $\eta = \eta_{opt}$.



In the shorter direction will then be

$$\alpha_{sh} = 1 - \frac{4}{3}\eta = 1 - \frac{4}{3} \cdot 0{,}214 = 0{,}715$$

$$m_{sh} = 0{,}715\,\frac{p\ell_{sh}^2}{8}$$

that is by 28,5 % smaller, then in case of not taken into account the effect of two-way ection. It is reasonable not to forget this rate, and try to apply in the practice when possible!