Lecture no. 6:

SHEAR AND TORSION
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I. Shear

1. Ways of modeling shear transfer in rc beams

1: the vault action
2: compression trajectories
3: tension trajectories
4: 1st shear crack, tension in the bottom reinforcement
5: 1st shear crack
6: elements of the shear reinforcement crossing the shear crack: links and bent-up bars
The truss model of Mörsch showing the way of transmitting shear to the support of simple supported rc beams

- lower chord: reinforcement equilibrating tension originated by flexure
- on top: concrete compression chord
- compressed concrete struts, with inclination angle $\theta$
- vertical tie-up forces absorbed by links

In the following the concrete compression strut inclination angle $\theta=45^\circ$ is considered for convenience in manual calculations.

In EC2 $1 \leq \cot \theta \leq 2.5$ is allowed, that is: $21.6^\circ \leq \theta \leq 45^\circ$
2. Absorbing shear in uncracked state

\[
\tau = \frac{SV}{bI_x}
\]

S: static moment  \( V \): shear force

\[
|\sigma_1| = |\sigma_2| = |\tau|
\]

strength rates:  \( f_{ct,d} = 0.1 \) unit  \( f_{cd} = 1 \) unit  \( \tau_d \approx 0.15 \) unit

consequence: tension failure occurs first: cracking parallel to \( \sigma_2 \)

Approximate shear resistance of the concrete section:

\[
V_{Rd,c} = c b_w f_{ct,d}
\]

c tabulated in DA
Values of $c$ for concrete grade C20/25

<table>
<thead>
<tr>
<th>$\rho_i$ [%]</th>
<th>$d$ [mm]</th>
<th>( \leq 200 )</th>
<th>300</th>
<th>400</th>
<th>500</th>
<th>600</th>
<th>700</th>
<th>800</th>
<th>900</th>
<th>1000</th>
</tr>
</thead>
<tbody>
<tr>
<td>C20/25</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>$f_{ctd} = 1,0$</td>
<td></td>
<td>0,00</td>
<td>0,429</td>
<td>0,371</td>
<td>0,338</td>
<td>0,316</td>
<td>0,301</td>
<td>0,288</td>
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<td>0,271</td>
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<tr>
<td>0,25</td>
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<td>0,429</td>
<td>0,371</td>
<td>0,340</td>
<td>0,325</td>
<td>0,314</td>
<td>0,305</td>
<td>0,298</td>
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<tr>
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<td>0,501</td>
<td>0,455</td>
<td>0,428</td>
<td>0,409</td>
<td>0,395</td>
<td>0,385</td>
<td>0,376</td>
<td>0,369</td>
<td>0,363</td>
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<tr>
<td>1,00</td>
<td></td>
<td>0,632</td>
<td>0,574</td>
<td>0,539</td>
<td>0,515</td>
<td>0,498</td>
<td>0,485</td>
<td>0,474</td>
<td>0,465</td>
<td>0,457</td>
</tr>
<tr>
<td>2,00</td>
<td></td>
<td>0,796</td>
<td>0,723</td>
<td>0,679</td>
<td>0,649</td>
<td>0,628</td>
<td>0,611</td>
<td>0,597</td>
<td>0,585</td>
<td>0,576</td>
</tr>
</tbody>
</table>
3. Ways of absorbing shear in cracked state

Neglected components:
- shear strength of the compression zone
- shear absorbed by friction along the shear cracks
- dowel action of bars of the tension reinf.

Shear equilibrated by links and bent-up bars:

\[ V_{Rd,s} = \frac{Z}{s} A_{sw} f_{ywd} \]

\[ V_{b} = (\sin \alpha + \cos \alpha) \frac{Z}{s_b} A_{sw,b} f_{ywd} \]

for \( \alpha = 45^\circ \):

\[ V_{Rd,s} = \sqrt{2} \frac{Z}{s_b} A_{sw,b} f_{ywd} \]
4. The maximum shear capacity limited by the compression strength of the concrete

\[ |\sigma_{c,\text{max}}| \approx \nu \cdot f_{cd} \]

Based on test results (in case of applying vertical links):

\[ V_{Rd,\text{max}} = 0.5 b_w z \nu f_{cd} \]

for vertical links + bent-up bars: 0.5→0.75

\[ z \approx 0.9d \] can be substituted

\[ \nu = 0.6 \left( 1 - \frac{f_{ck}}{250} \right) \] effectiveness factor
5. Design condition of the shear capacity

\[
V_{Rd} = \min \left\{ V_{Rd,\text{max}} \right\} \geq V_{Ed}
\]

\[
\max \left\{ V_{Rd,c}, \right. \left. V_{Rd,s} = V_{Rd,s}^{\ell} + V_{Rd,s}^{b} \right\} \geq V_{Ed}
\]

it is to be respected that: \( V_{Rd,s}^{\ell} \geq 0.5V_{Ed} \)
6. The practical way of shear design

If $V_{Ed} \geq V_{Rd,c}$ and no bent-up bars are used, set diameter of vertical links, and calculate the necessary spacing of links:

$$s_s = \frac{0.9 d A_{sw} f_{ywd}}{V_{Ed}}$$

if bent-up bars are used:

$$s_s = \frac{0.9 d A_{sw} f_{ywd}}{V_{Ed} - V_{Rd,c}^b}$$

where $V_{Ed} < V_{Rd,c}$

Different links:

- min. links can be used (see DA)

Dashed area: shear to be equilibrated by shear reinforcement
7. Special problems in shear design

Variable height of the beam

a) variation on side of the compression zone

\[ V_{Ed} = V_{Ed} - N_c \tan \alpha \]

b) variation on side of the tension zone

\[ V_{Ed} = V_{Ed} - N_s \tan \alpha_s \]

Tieing-up concentrated load or secondary beam

- by bent-up bar

\[ A_{sw}^{b} \]

1,2: local safety factor

- by vert. links

\[ 45^\circ \]

\[ \Sigma A_{sw} \]

links of principal beam

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8. The short cantilever

Force to equilibrate by the main bars:

\[ F_s \cong F_{Ed} \left( \frac{a_c}{z_0} + 0.1 \right) \]

\[ 45^\circ \leq \theta \leq 68^\circ \]
9. Check of the beam end

The force to be absorbed by the tension reinforcement at the beam end:

\[ \sum M_c = 0 : \rightarrow F_{Ed} \]

Formulae for \( F_{Ed} \) see in DA:

<table>
<thead>
<tr>
<th>Values of the tensile force ( F_{Ed} )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>There is no designed shear reinforcement</strong></td>
</tr>
<tr>
<td>. ( \theta = 45 ) crack inclination angle</td>
</tr>
<tr>
<td>links</td>
</tr>
<tr>
<td>( \left(1.1 + 1.1 \frac{a_i}{d}\right)V_{Ed} )</td>
</tr>
<tr>
<td>( \left(0.25 + 1.1 \frac{a_i}{d}\right)V_{Ed} )</td>
</tr>
</tbody>
</table>

Approximation: \( \approx h/2 = (d/0.85)/2 \):

| \( 1.75|V_{Ed}| \) | \( 1.15|V_{Ed}| \) | \( 0.9|V_{Ed}| \) |

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10. Reduction of the anchorage length by 90° bents and hooks

Anchorage of tension bars at the beam end is problematic due to lack of space. Solution: use of hooks, bends, loops, welded anchorage devices.

\[ \ell_{b,\text{red}} = \alpha_a \ell_b \]

<table>
<thead>
<tr>
<th>Hook</th>
<th>90° bend</th>
<th>Loop</th>
<th>( \alpha_a )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \geq 5\phi ) ( \geq 150^\circ ) ( \phi ) ( l_{b,eq} )</td>
<td>( \geq 5\phi ) ( 90^\circ \leq \alpha &lt; 150^\circ ) ( \phi ) ( l_{b,eq} )</td>
<td>( l_{b,eq} )</td>
<td>0.7 ( ^1 )</td>
</tr>
<tr>
<td>welded transverse bar within ( l_{bd} )</td>
<td></td>
<td></td>
<td>0.7</td>
</tr>
</tbody>
</table>

\^1 The reduction is valid only if along the bent portion the concrete cover in direction perpendicular to the plane of bending is \( >3\phi \), transverse compression is acting, and links are used, otherwise \( \alpha_a = 1.0 \).
11. Parallel shifting of the moment diagram due to diagonal shear cracks

Reason of shifting: inclination of shear cracks

Direction of shifting $M$:

Extent of shifting: $a = \begin{cases} 
z = 0.9d \\
0.5z = 0.45d \\
0.25z = 0.225d 
\end{cases}$

- no shear reinf.
- shear reinforcement: links
- shear reinf.: links+bent-up bars
12. Constructional rules of links and bent-up bars

At least half of the shear force should be equilibrated by links. The shear steel ratio: \( \rho_w = A_{sw} / (s \cdot b_w \cdot \sin \alpha) \)

\[ \rho_{w,\text{min}} = (0,08 \sqrt{f_{ck}}) / f_{yk} \]

is tabulated in DA:

<table>
<thead>
<tr>
<th>( f_{yk} )</th>
<th>C12/16</th>
<th>C16/20</th>
<th>C20/25</th>
<th>C25/30</th>
<th>C30/37</th>
<th>C35/45</th>
<th>C40/50</th>
<th>C45/55</th>
<th>C50/67</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>1,00</td>
<td>1,00</td>
<td>1,00</td>
<td>1,00</td>
<td>1,00</td>
<td>1,00</td>
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<tr>
<td>400</td>
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<td>1,00</td>
<td>1,10</td>
<td>1,18</td>
<td>1,26</td>
<td>1,34</td>
<td>1,41</td>
</tr>
<tr>
<td>240</td>
<td>1,15</td>
<td>1,33</td>
<td>1,48</td>
<td>1,67</td>
<td>1,81</td>
<td>1,95</td>
<td>2,05</td>
<td>2,21</td>
<td>2,33</td>
</tr>
</tbody>
</table>

Maximum spacing of links \( s_{l,\text{max}} = 0,75d \)

In case of designed compression steel \( s_l \leq 15 \phi' \)

Maximum spacing of 45° bent-up bars \( s_{b,\text{max}} = 1,2d \)
13. Shear transmitted by diagonal compression to the support

This is possible according to EC2, but will be neglected for simplification as a safe approximation.
II. Torsion

1. Way of handling of torsion in design practice

**Try to avoid torsion if possible!**

Example

\[ T_{\text{max}} = \frac{pl_c e l}{2} \]

Section of construction a)

The lintel is subjected to \( T_{\text{max}} \) at the support.

Moments of the balcony slab are equilibrated by the joining inside monolithic rc slab. The lintel is not subjected to torsion!
2. The behaviour of rc beams subjected to torsion

diagonal cracking continuing along all the four sides

Both longitudinal bars and links are intersecting the cracks: they both work in equilibrating torsion

Due to diagonal cracking the rigidity of the member (beam) is much reduced. The resistance to flexural deformations is decreasing significantly by the effect of torsion.
3. The shear flow equilibrating torsion along the perimeter of the section

\[ T = 2qhb = q2A_c \]

Let us express the shear flow \( q \) by capacities of links and longitudinal bars!
4. Torsional moment capacity due to links

The torsional moment capacity due to links by substituting the expression obtained for $q$:

$$T_{Rd,s} = \frac{A_{sw}f_{ywd}}{s_s} 2A_c$$
5. Torsional moment capacity due to longitudinal bars

The tensile force to be equilibrated by longitudinal bars:

\[
\Sigma H = 2qh + 2qb = qP_c = A_{sl}f_{yd}
\]

here \(2h + 2b = P_c\) is the perimeter measured along centerline of links

Expressing \(q\):

\[
q = \frac{A_{sl}f_{yd}}{P_c}
\]

and substituting \(q\) in \(T\), the torsional moment capacity due to longitudinal bars:

\[
T_{Rd,A_{sl}} = \frac{A_{sl}f_{yd}}{P_c} 2A_c
\]
6. The torsional moment capacity of rc beams

\[
T_{Rd} = \min \left\{ T_{Rd, s}, T_{Rd, A_{sl}} \right\} \leq T_{Rd, \text{max}} = \psi_{cd} A_c t_{eff}
\]

Here \( t_{eff} = \max \left[ \frac{A_c}{P_c}, 2a \right] \)

Uniformly distributed – closed - links and longitudinal bars should be designed, independently from links iand longitudinal bars designed for shear and moment.

Closed links to be designed for torsion