Budapest University of Technology and Economics

Department of Mechanics, Materials and Structures

English courses

Reinforced Concrete Structures

Code: BMEEPSTK601

Lecture no. 4:

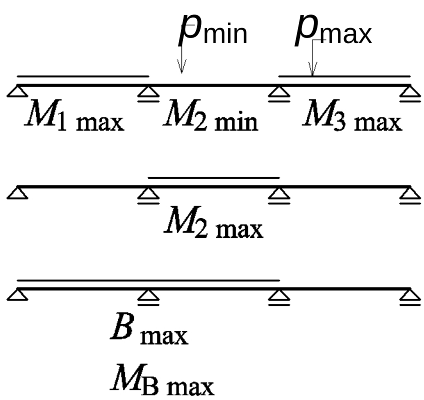
# Plastic design of continuous beams

Content:

## 1. Methods of determination of the moment distribution in continuous beams

The moment distribution can be determined by applying *elastic methods* (moment distribution method, tabulated formulae, force method etc.), or *plastic methods* (for example plastic limit of equilibrium).

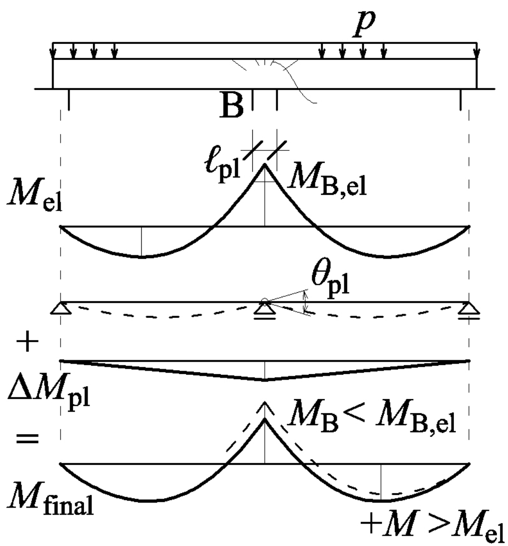
For section design we need *extreme values* of moments and shear forces.When using elastic methods, the extreme values are determined from different *loading scheemes*:




$p_{min}=g_d$
$p_{max}= g_d+q_d$

## 2. Plastic moment redistribution in continuous beams

Reason of moment redistribution: plastic rotation occures above intermediate supports:




Plastic curvatures along $l_{pl}$ concent-rated in rotation of the plastic hinge above support B:

$$\theta_{pl} = \rho_{mean} l_{pl}$$

$\Delta M_{pl}$: redistribution moments

Precondition of moment redistribution: the rotation capacity without failure of the concrete.

To increase the *deformation capacity*: $x_c$ at plastic hinges (above intermediate supports) must be limited.

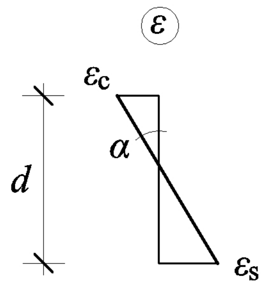
Difference in plastic behavior of rc slabs and beams:

Slabs: $\xi_c = x_c/d$ is small→ deformability, plastic rotation capacity is greater

Beams: $\xi_c = x_c/d$ is greater→ deformability, plastic rotation capacity is limited, may be necessary to check by moment redist-tribution (or limitation of $x_c$ is necessary)

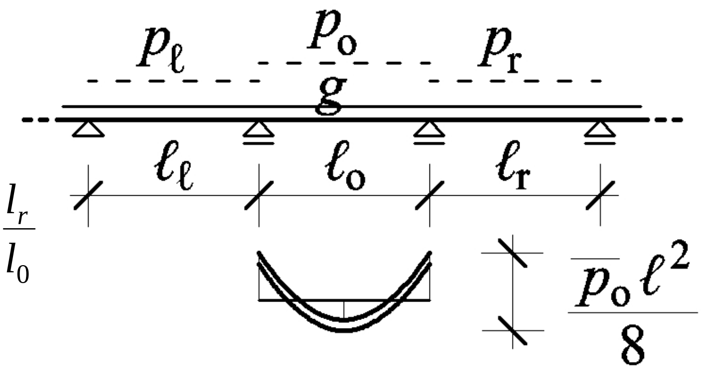
Curvature: $\rho = \tan\alpha = \dfrac{\varepsilon_c + \varepsilon_s}{d}$

If $x_c$ is smaller at rupture ($\varepsilon_c$=3,5‰), the curvature is greater



# 3. **The simplified plastic analysis of continuous beams proposed by Steven Menyhárd in the 1950-ies: the method of *substitutive loading***

$$p'_0 = g + q_0\left[1 + \frac{1}{4}(m_l t_l^2 + m_r t_r^2)\right]$$




$$m_l = \frac{q_l}{q_0} \qquad m_r = \frac{q_r}{q_0} \qquad t_l = \frac{l_l}{l_0} \qquad t_r = \frac{l_r}{l_0}$$

If 0,8 ≤ $t_l$, $t_r$ , $l_l$, $l_r$ ≤ 1,25, then:

$m_l\, t_l^2$=1 and $m_r\, t_r^2$=1 can be substituted:

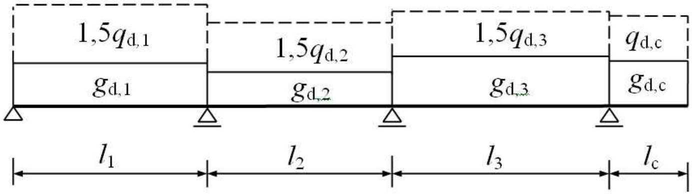
o

$$p'_0 = g + 1{,}5 q_0$$

*Value of the substitutive loading:* self-weight plus variable load increased by 50% in all spans* and total loading on cantilevers, that is:

$$p_{di} = g_{d,i} + 1{,}5q_{di} \quad \text{on the spans}$$

$$p_{d} = g_{d} + 1{,}0q_{d} \quad \text{on cantilever(s)}$$




In the above expressions: $g_{\mathrm{d,i}} = \gamma_{\mathrm{G,i}} g_{\mathrm{ki}}$ and $q_{\mathrm{d,i}} = \gamma_{\mathrm{Q,i}} q_{\mathrm{ki}}$ .

When *substitutive loading* is applied, *no loading schemes* are to be investigated

* According to the Hungarian Standard, in case of beams with monolithic joints at supports 25% increase is enough

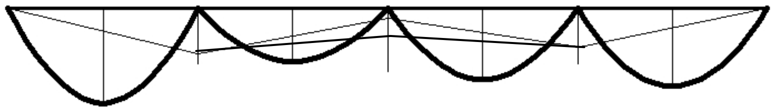
*Conditions to be fulfilled*:

- the beam is supported by hinged supports and loaded by uniformly distributed load
- $0,8 \leq l_i / l_{i+1} \leq 1,25$ where $l_i$ and $l_{i+1}$ are any two neighbouring spans
- $0,8 \leq p_{di} / p_{d,i+1} \leq 1,25$ where $p_{d,i} = g_{d,i} + q_{d,i}$
- $q_{di} \leq 2 g_{di}$

The procedure below can only be applied if the plastic deformation capacity of the sections above intermediate supports is assured.

| Limits of $\xi_c$ | way of checking the rotation capacity |
|---|---|
| $\xi_c \leq 0,2$ | no check needed(case of slabs in general) |
| $\xi_c \leq 0,36$ | simplified check |
| otherwise | detailed check is necessary |

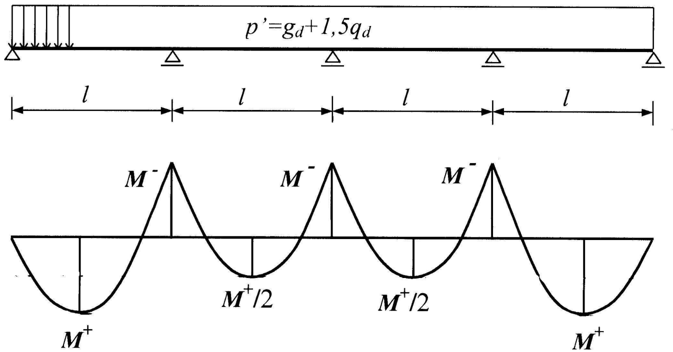
## 4. **Another simple way of determining moment distribution in continuous beams**



$$\frac{p_1^{'} \ell_1^2}{8}$$

## 5. Specific case: equal spans, constant uniformly distributed load

Moment distribution in case of equal spans *l* and throughout constant uniformly distributed load (without cantilever)




Value of the substitutive loading: : $p'=g_d+1{,}5q_d$

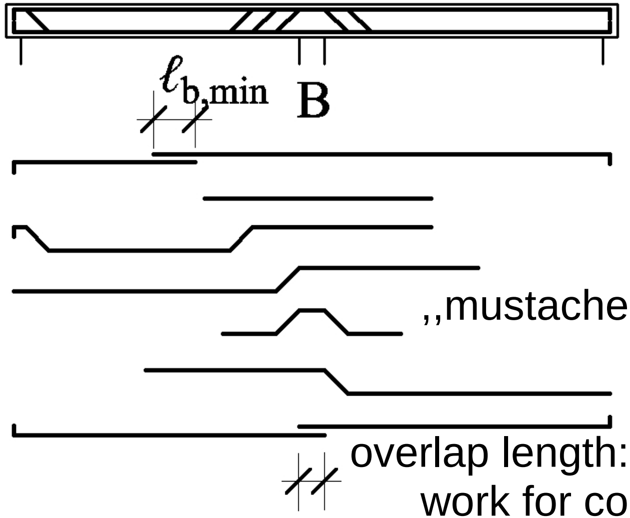
Moments in the first and last spans and above intermediate supports:

$$M^+ = |M^-| = p_d' l^2/11{,}6$$

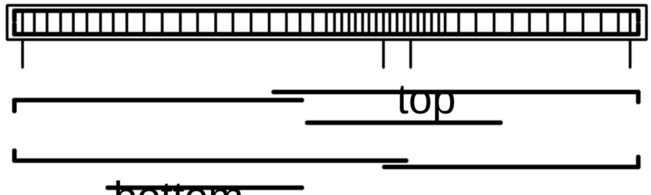
Positive moments in the interior spans: $M^+ =$ M$^+/2 = p_d' l^2/23{,}2$

## 6. **Reinforcement systems used by detailing of continuous beams**

a) Use of bent-up bars



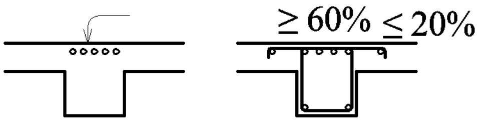

b) Straight bars + links



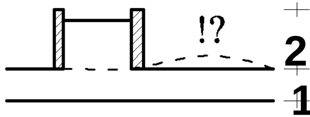
# 7. Some special design problems by detailing of continuous beams

**little gaps between bars**

**concreteing?**




**solution**

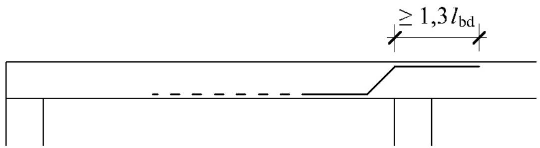


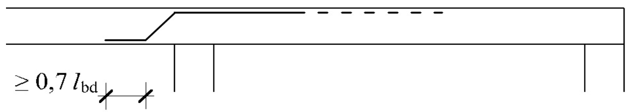

**2nd phase**

**1st phase**

**of concreting**

## Anchorage length of bent-up bars in tension and compression zone respectively:

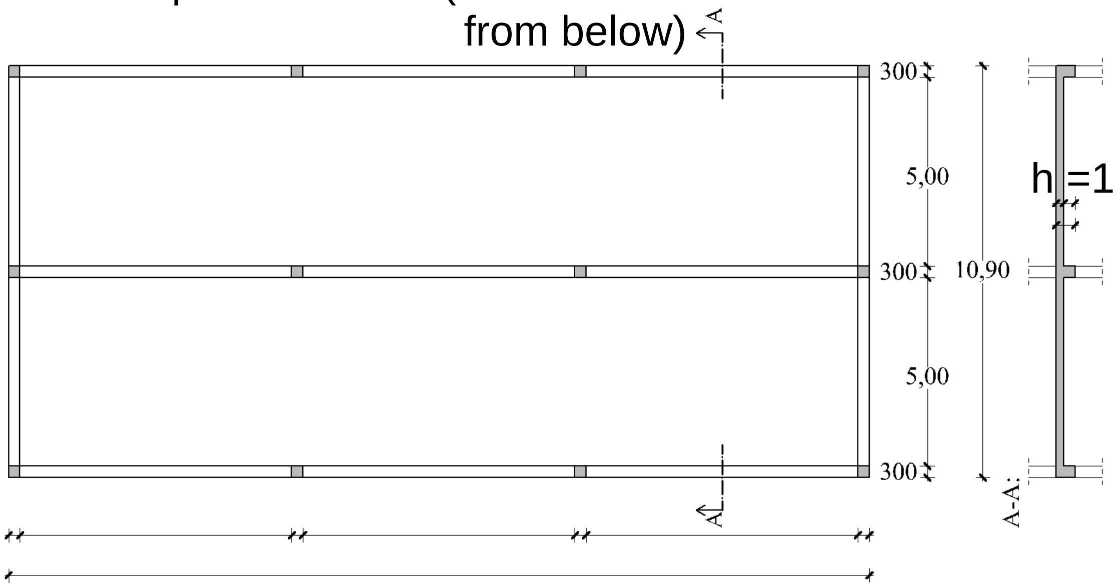


## 8. Numerical example

Determine the design value of the moments of the floor slab given on the plan below, if $g_k$= 8 kN/m$^2$ and $q_k$= 3 kN/m$^2$!

Stuctural plan of a floor (lookout of the floor construction as seen from below)



Solution:

$$l_{eff} = 5{,}00 + 2\frac{0{,}18}{2} = 5{,}18 \text{ m}$$

$$p_d = 1{,}35 \cdot 8 + 1{,}5 \cdot 3 = 10{,}8 + 4{,}5 = 15{,}3 \text{ kN/m}^2$$

$$p'_d = 10{,}8 + 1{,}5 \cdot 4{,}5 = 17{,}55 \text{ kN/m}^2$$

$$M_{Ed,B} = \frac{p'_d l_{eff}^2}{11{,}6} = \frac{17{,}55 \cdot 5{,}18^2}{11{,}6} = 40{,}6 \text{ kNm}$$

$M_{\text{Ed,span}} = M_{\text{Ed,B}} = 40{,}6$ kNm