Budapest University of Technology and Economics

Department of Mechanics, Materials and Structures
English courses
Reinforced Concrete Structures
Code: BMEEPSTK601

Lecture no. 12:

REINFORCED CONCRETE COLUMNS, BUCKLING

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Introduction

Manual calculation of rc. sections subjected to eccentric compression becomes very simple by using the linearized M_R-N_R capacity diagram. The exact solution of the equilibrium conditions would have been rather complicated, because steel bars are often in the elastic range at rupture of the extreme concrete fibre in the ultimate limit state. This would imply solution of higher degree equation systems, so that manual handling of the problem is rather complicated.

Check and design of rc. columns subjected to eccentric compression is difficulted mainly through the necessity of taking second order effects into consideration, that is the danger of loss of stability due to buckling. The problem of respecting second order eccentricity increment will be overcome by the use of tabulated specific values of it, which had been numerically determined for different slenderness ratios.

Even manual design by succesive checks or control of computerized design calculations can be done by the use of this method.

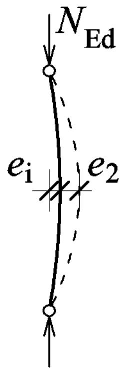
1. Axially loaded rc. columns

As already mentioned earlyer, the Eurocode 2 does not make difference between axial and eccentric compression.

e_i: eccentricity due to imperfection

(way of determination see later by eccentric compression)

e_2: 2nd order effect (deformation)



$$M = N_{Ed}\,(e_i + e_2)$$

a) Solution of the problem by method of strength of materials for elastic behavior:

$$M_o = N_{Ed} e_i$$

$$M = \psi M_o \quad \text{where} \quad \psi = \frac{1}{1 - \frac{N_{Ed}}{N_{cr}}} \qquad N_{cr} = \frac{\pi^2 EI}{\ell_o^2} \quad (\text{that is: } e = e_i + e_2 = \psi e_i)$$

b) Solution of the problem according to our design aids

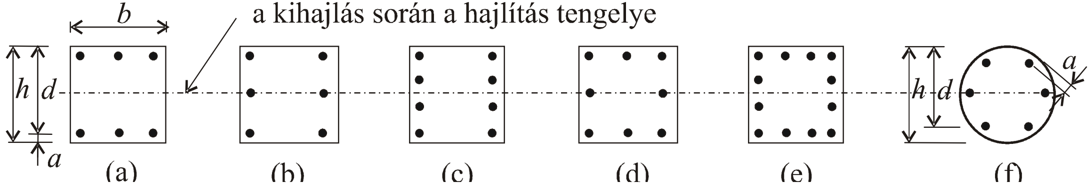
Our design aids formally maintains the traditional simple way of handling of axial compression (as it was done according to the Hungarian national standard), but respecting – through background calculations – the above mentioned unified way of handling of axial and eccentric compression.

A reduction coefficient φ is used – in function of the slenderness ratio of the column and the intensity of its longitudinal reinforcement – to respect the effect of imperfection and the second order deformation:

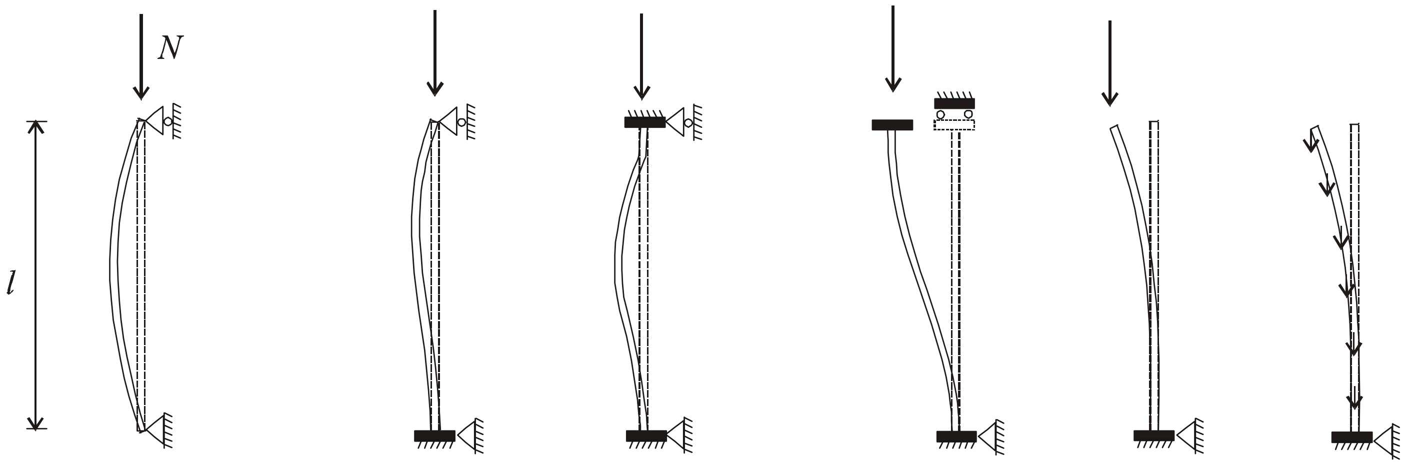
$N_{Rd} = \varphi N'_u$ where $N'_u = A_c f_{cd} + A_s f_{yd}$

Values of the reduction coefficient φ in function of the concrete strength and the slenderness ratio $\alpha=l_0/h$

Concrete	Rectangular cross-section with bars arranged in two rows (Fig. a)						Rectangular cross-section with bars arranged in three rows (Fig. b)						
	$\alpha=l_0/h$						$\alpha=l_0/h$						
	≤12	14	16	18	20	22	≤10	12	14	16	18	20	22
C20/25	0,86	0,81	0,75	0,68	0,56	0,39	0,88	0,83	0,77	0,68	0,54	0,41	0,33
C25/30	0,86	0,80	0,74	0,65	0,49	0,38	0,88	0,83	0,76	0,65	0,46	0,40	0,32
C30/37	0,85	0,80	0,74	0,62	0,42	0,37	0,88	0,83	0,76	0,64	0,43	0,37	0,32
C35/45	0,85	0,80	0,73	0,59	0,41	0,35	0,88	0,83	0,76	0,62	0,42	0,36	0,30
C40/50	0,85	0,80	0,73	0,56	0,39	0,33	0,88	0,83	0,75	0,61	0,41	0,34	0,28
C45/55	0,85	0,80	0,72	0,54	0,39	0,31	0,88	0,83	0,75	0,59	0,40	0,33	0,26
C50/60	0,85	0,79	0,71	0,51	0,38	0,30	0,88	0,83	0,75	0,58	0,40	0,32	0,24

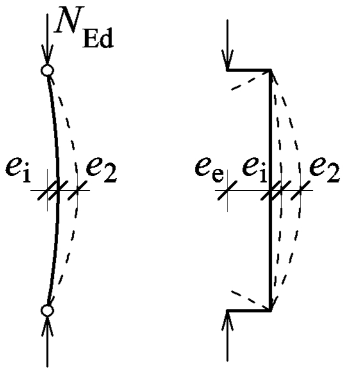


2. The effective length of individual columns



(a) $l_0=l$ (b) $l_0=0.7l$ (c) $l_0=0.5l$ (d) $l_0=l$ (e) $l_0=2l$ (f) $l_0=1.2l$

3. Eccentrically loaded rc. columns



The total eccentricity to be considered is:

$$e = \max \begin{cases} e_e + e_i + e_2 & \text{sum of the eccentricities} \\ M_{02}/N_{Ed} & \text{eccentricity at the end of the column} \\ e_0 & \text{minimum eccentricity} \end{cases}$$

Determination of eccentricities according to Eurocode 2

-Eccentricity due to applied moment:

$$e_e = \frac{M_{0e}}{N_{Ed}}$$

In case of *non-sway frames*:

$$M_{0e} = \max\begin{cases} 0.6M_{02} + 0.4M_{01} \\ 0.4M_{02} \end{cases} \qquad \left(|M_{02}| > |M_{01}|\right)$$

In case of *sway frames*:

$$M_{0e} = M_{02} \qquad \left(|M_{02}| > |M_{01}|\right)$$

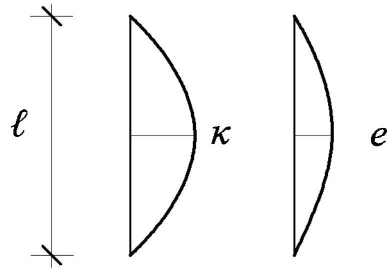
-Eccentricity due to initial curvature (imperfection):

$$e_i = \begin{cases} \dfrac{\ell_o}{400} & \text{if } \ell \leq 4\text{m} \\ \dfrac{2}{\sqrt{l}} \dfrac{\ell_o}{400} & \text{if } 4\,\text{m} \prec \ell \prec 9\,\text{m} \\ \dfrac{2}{3} \dfrac{\ell_o}{400} & \text{if } \ell \geq 9\,\text{m} \end{cases}$$

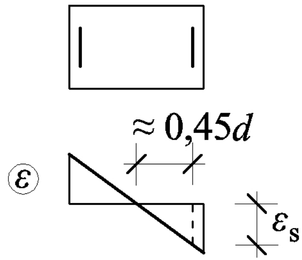
-Eccentricity caused by second order moment:

$e_2 = \dfrac{1}{r} \dfrac{l_o^2}{\pi^2} \approx \dfrac{1}{r} \dfrac{l_o^2}{10}$, where:

$\dfrac{1}{r} = K_r K_\varphi \dfrac{1}{r_o}$ the curvature



$$\frac{1}{r_o} = \frac{f_{yd} / E_S}{0.45\, d'}$$ the initial curvature



$$K_\varphi = \max\{1 + \beta\varphi_{ef}\ ;1\}$$ effect of creep

$$\beta = 0.35 + \frac{f_{ck}}{200} - \frac{\lambda}{150};$$ λ is the slenderness ratio of the column, f_{ck} should be substituted in N/mm²

$$K_r = \min\left\{\frac{N'_u - N_{Ed}}{N'_u - N_{bal}};1\right\}$$ effect of the normal force,

$$(N'_u = f_{cd}bh + A_s f_{yd}),$$

$$\lambda = \frac{l_o}{h}\sqrt{12}$$

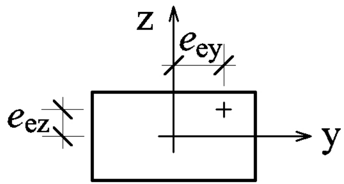
4. Determining additional eccentricities by the use of design aids tables

Specific values of additional eccentricities												
	l_o/d_1											
	0	6	8	10	12	14	16	18	20	22	24	26
e_i/d_1	0,000	0,015	0,020	0,025	0,030	0,035	0,040	0,045	0,050	0,055	0,060	0,065
e_2/d_1	0,000	0,034	0,058	0,085	0,116	0,151	0,189	0,229	0,271	0,313	0,355	0,395
$(e_i + e_2)/d_1$	0,000	0,049	0,078	0,110	0,146	0,186	0,229	0,274	0,321	0,368	0,415	0,460
Specific values of additional eccentricities *(continued)*												
	l_o/d_1											
	28	30	32	34	36	38	40	42	44	46	48	50
e_i/d_1	0,070		0,080	0,085	0,090	0,095	0,100	0,105	0,110	0,115	0,120	0,125
e_2/d_1	0,434	0,471	0,526	0,594	0,666	0,742	0,823	0,907	0,995	1,088	1,184	1,285
$(e_i + e_2)/d_1$	0,504	0,546	0,606	0,679	0,756	0,837	0,923	1,012	1,105	1,203	1,304	1,410

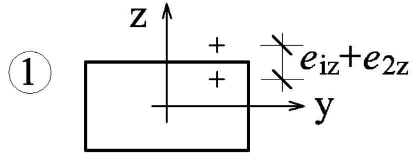
The use of these tabulated eccentricities replaces the use of the formulas given above for determination of e_i and e_2

5. Two independent checks

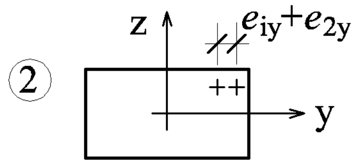
Initial eccentricities due to the applied moments $M_{Ed,z}$, $M_{Ed,y}$ and axial force N_{Ed}



1st investigation: additional eccentricities parallel to the *z*-axis

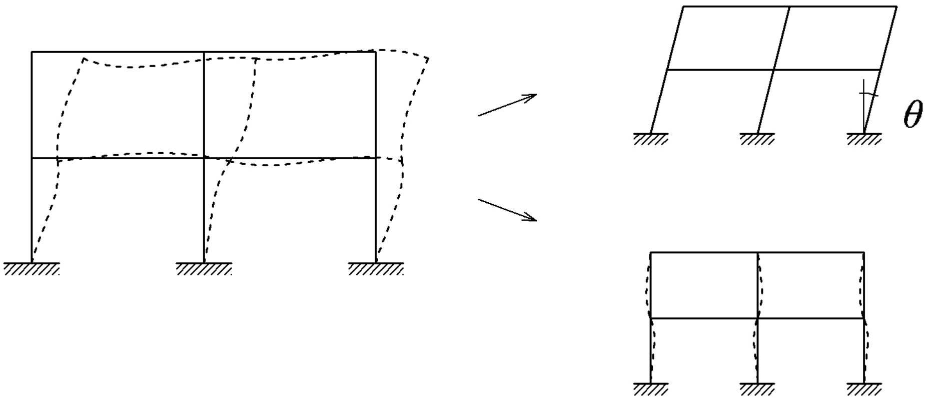


2nd investigation: additional eccentricities parallel to the y-axis

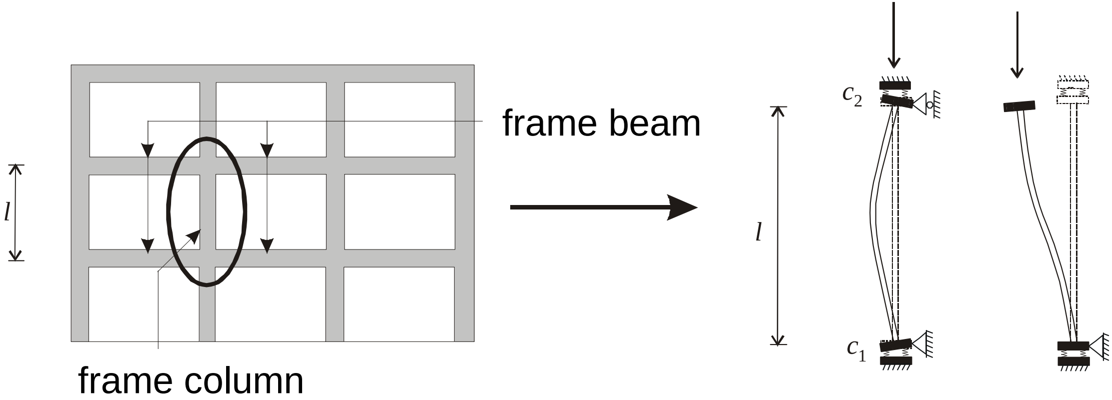


6. Effective length of columns in frames

The effect of inclination – imperfection – and that of curvateure (second order effect) is considered separately:



Determination of the effective length ℓ_o

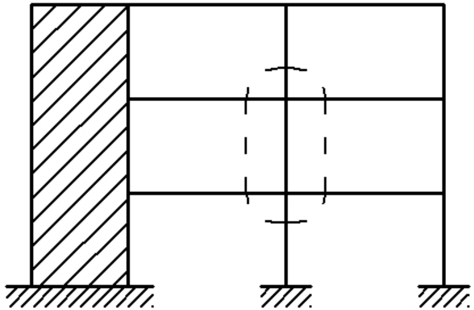


non-sway frame $0{,}5\ell \leq \ell_o \leq \ell$

sway frame $\ell_o > \ell$

For non-sway frames:

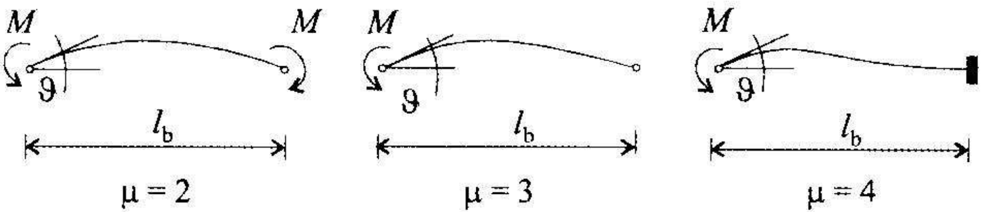
$$l_o = 0.5l\sqrt{\left(1+\frac{k_1}{0.45+k_1}\right)\left(1+\frac{k_2}{0.45+k_2}\right)}$$



where *k* is the relative flexibility of the stabilizing bars (indices 1and 2 refer to the ends of the column):

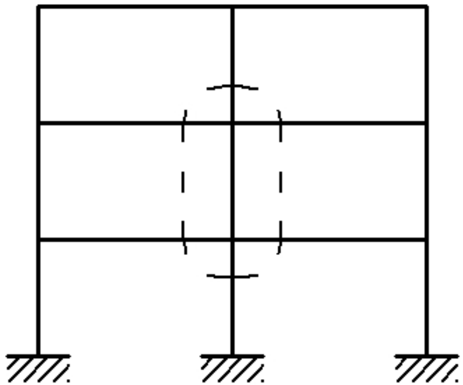
$$k = \frac{1/c}{l/EI} = \frac{EI/l}{c} \qquad c = \Sigma\mu\frac{EI_b}{l_b}$$

rotational rigidity coefficient of joining beams:



For sway frames:

$$l_o = \max\begin{cases} l\sqrt{\left(1+10\dfrac{k_1k_2}{k_1+k_2}\right)} \\ l\left(1+\dfrac{k_1}{1+k_1}\right)\left(1+\dfrac{k_2}{1+k_2}\right)\end{cases}$$



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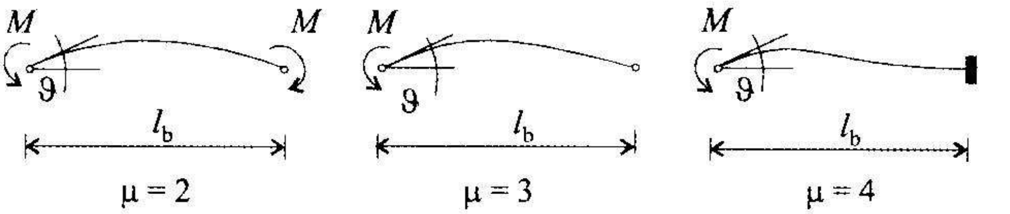
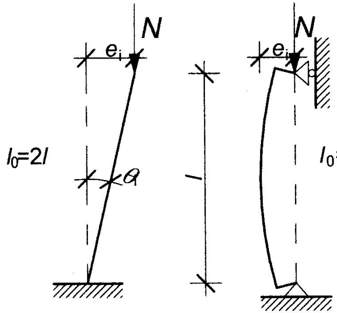


7. Ways of considering the effect of inclination due to construction imperfection

a) *Eccentricity* due to inclination of the column axis of separated columns:

$$e_i = \theta \frac{\ell_o}{2} = \tan\theta \frac{\ell_o}{2} = \frac{1}{200} \cdot \frac{\ell_0}{2} = \frac{\ell_o}{400}$$

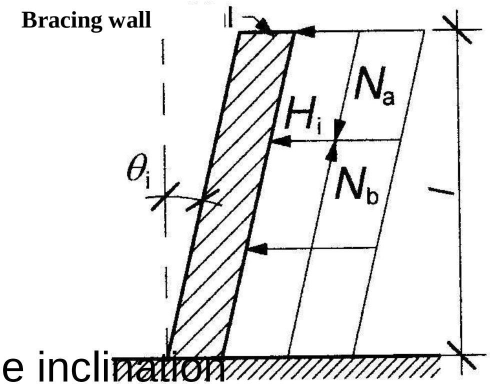
(see page 10)



un-braced braced column

b) *Additional horizontal force* acting at each level of multi-storey buildings caused by inclination due to imperfection.

These horizontal forces should be applied to the bracings of braced buildings, and to the rigid frame of un-braced structures.



The following safe approximation can be applied:

$$H_i = (N_b - N_a)/200,$$

where N_a and N_b are internal forces

More exact calculation:

$$H_i = \theta_i \cdot (N_b - N_a)$$

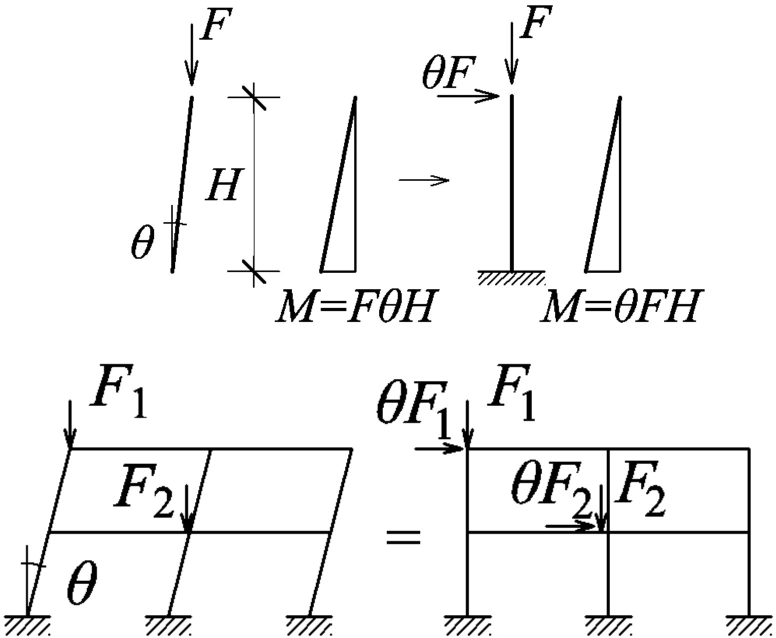
$$\theta_i = \alpha_n \cdot \alpha_m \; \theta_0$$

$\theta_0 = 1/200$, is the basic value of the inclination

$\alpha_n = 2/\sqrt{l}$ $\quad 2/3 \leq \alpha_n \leq 1$, where l is the height of the building in m

$\alpha_m = \sqrt{0{,}5(1+1/m)}$, where m is the number of columns on one level

Respecting the inclination due to imperfection or application of horizontal forces at joints which produce the same additional moments, constitute two alternatives of the same effect.



8. Constructional rules of columns

Reinforced concrete columns are linear members, having cross sectional side length ratios $h/b \leq 4$.

Minimum side length of solid column sections

for columns concreted in vertical position: $b_{min} \geq 200$ mm,

for columns concreted in lying position : $b_{min} \geq 120$ mm.

Rules concerning the longitudinal reinforcement

Minimum bar diameter: $\phi_{min} = 8$ mm

Minimum steel area $A_{s,min} = \max(0{,}1 N_{Ed}/f_{yd}\ ;\ 0{,}003 A_c)$

Maximum steel area: $A_{s,max} = 0{,}04 A_c$, which can be doubled in section of overlap.

Maximum spacing between elements of the reinforcement (s)

Longitudinal reinforcement

Column sections composed of rectangles, polygonal sections:

$s \leq 400$ mm and at least one bar in each corner

-general cross sectional forms composed of rectangles and rectangles with $h > 400$ mm or polygonal sections: $s \leq 300$ mm and at least one bar in each corner

-rectangular sections if $h \leq 400$ mm:

at least one bar in each corner

-circular column section: at least 6 pieces* of longitudinal bars [DIN and Hungarian rc standard] and $s \leq 300$ mm

At intersection points of link legs longitudinal bars must be applied

Overlap length of the longitudinal bars of axially loaded columns: l_{bd}.

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* Eurocode allows use of 4 bars only

Maximum link spacing[*]*:*

$$s_{w,max} = \min \begin{Bmatrix} 15\phi_{min} \\ h_{min} \\ 400\,\text{mm} \end{Bmatrix}$$

ϕ_{min} is the smallest diameter of longitudinal bars, h_{min} is the smallest side length

The *link diameter* is at least ¼ times the diameter of the maximum diameter of the longitudinal bars: $\phi_l \geq \frac{\phi}{4}$, but minimum 6 mm.

Link spacing should be increased near the introduction point of the load, and at breakpoints of longitudinal bars.

A reduction factor equal to 0,6 should be applied for the minimum spacing of links along a distance equal to the greater side length of the column: above and below of joining beams and slabs and along the overlap length of longitudinal bars of diameter $>\phi 14$ mm.

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[*] The Eurocode proposes $20\phi_{min}$ for the greatest link spacing, the prescription of $12\phi_{min}$ is stricter (DIN 1045-1, 2001 and Hungarian rc. standard), we also recommend the latter one.

At breakpoints of longitudinal bars, if the rate of the break is >1/12, links must be designed for the horizontal component.
At beam-column joints links of the beam must be interrupted (see figure).
Fixing of longitudinal bars:

All bars should be fixed against horizontal displacement. Corner bars can be considered fixed by links.

In the compression zone of the cross section no bar can be more distantiated from a fixed bar than 150 mm. Use of extra links may be necessary.