REINFORCED CONCRETE SECTIONS SUBJECTED TO AXIAL AND ECCENTRIC COMPRESSION
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Introduction
The Eurocode 2 does not distinguish axial and eccentric compression, because due to imperfection, the theoretically axial compression force does always have some eccentricity.
1. Suppositions

-plane sections remain plane
-bound connection between concrete and steel is perfect
-the mechanical behavior of concrete and steel can be modelled by the idealized $\sigma$-$\varepsilon$ relationships indicated below:
2. Failure modes

Failure modes of the rc. section subjected to axial or eccentric compression can be given by the $\varepsilon$-diagrams given below.

The deformation of the steel at rupture of the concrete extreme fibre is

$$\varepsilon_s < \varepsilon_{s1} = \frac{f_{yd}}{E_s}$$

that is steel is in the elastic state:

$$\sigma_s = \frac{560}{\xi_c} - 700 \text{ (N/mm}^2)$$

For practical design the use of $M_R-N_R$ capacity curves is recommended!
3. The $M_R-N_R$ capacity diagram of concrete sections

For concrete sections the $M_R-N_R$ capacity curve has the character indicated below. Because of zero tensile strength of the concrete, at $N_R=0$, the capacity moment $M_R=0$. 
4. The $M_R-N_R$ capacity diagram of reinforced concrete sections

Point 1: axial compression

\[ N_{Rd} = A_c f_{cd} + A_s f_{yd} = N_u \]
\[ M_{Rd} = 0 \]
Point 2: eccentric compression \((x_c=x_{co})\)

\[
N_{Rd} = b x_{co} f_{cd} = N_{bal}
\]

\[
M_{Rd} = N_{bal} \left( \frac{h}{2} - \frac{x_{co}}{2} \right) + A_{s1} f_{yd} z_s = \Delta M_R + M_s = M_{Rd,max}
\]
Point 3: flexure

\[ N_{Rd} = 0 \]

\[ M_{Rd} \approx A_{s1} f_{yd} z_s = M_s \]
5. Safe and unsafe applied force combinations

![Diagram showing safe and unsafe force combinations with points labeled 1, 2, and 3.]

- Cases of small eccentricities
- Great eccentricities

Unsafe force combination: \((M_{Ed2}, N_{Ed2})\)

Safe force combination: \((M_{Ed1}, N_{Ed1})\)
6. Applied force combinations to be investigated

Any of the below force combinations can be dangerous, should be investigated:

1. \((M_{Ed,\text{max}}, N_{Ed,\text{min}}(M_{Ed,\text{max}}))\)

2. \((M_{Ed,\text{max}}, N_{Ed,\text{max}}(M_{Ed,\text{max}}))\)

3. \((N_{Ed,\text{max}}, M_{Ed,\text{max}}(N_{Ed,\text{max}}))\)

4. \((N_{Ed,\text{min}}, M_{Ed,\text{max}}(N_{Ed,\text{min}}))\)
7. Linearization of the $M_R-N_R$ capacity diagram
8. Axial and eccentric tension

Part of the capacity diagram below the $M_R$ axis show capacity of the rc section in eccentric and axial tension.

For axial tension:

$$N_{Rd,t} = (A_{s1} + A_{s2})f_{yd}$$
9. Use of a series of dimensionless capacity diagrams

\[ \mu = \frac{A_s f_{yd}}{bhf_{cd}} \]

\[ m = \frac{M_{Ed}}{bh^2 f_{cd}} \]

\[ n = \frac{N_{Ed}}{bhf_{cd}} \]

\[ A_s = \sum A_{si} = 2A_{s1} \]
10. $M_R-N_R$ capacity diagram of nonsymmetric reinforced concrete sections
11. The \((M_{Rz}, M_{Ry}, N_R)\) capacity body
12. **Check of rc. sections subjected to compression with double eccentricity**

The given force combination is safe!
The given force combination is unsafe!
13. Numerical example

The column section given below is subjected to an eccentric compression force $N_{Ed} = 1620$ kN, $e = 27.7$ mm. Check the section by determining the simplified (linearized) $M_R - N_R$ capacity diagram of it!

Materials: concrete: C20/25-32/KK steel: B60.50
Concrete cover 20 mm link diameter: $\phi_k = 8$ mm

\[
\begin{align*}
  f_{cd} &= \frac{20}{1.5} = 13.3 \text{ N/mm}^2 \\
  f_{yd} &= \frac{500}{1.15} = 435 \text{ N/mm}^2 \\
  \xi_{co} &= 0.49
\end{align*}
\]
Solution:

\[ d = 350 - 20 - 8 - 8 = 314 \text{ mm} \]

\[ N'_u = bh_f cd + A_s \cdot 435 = (350^2 \cdot 13,3 + 1608 \cdot 435) \cdot 10^{-3} = \]
\[ = 1633 + 700,3 = 2333,3 \text{ kN} \]

\[ N_{bal} = b \xi_{co} df_{cd} = 350 \cdot 0,49 \cdot 314 \cdot 13,3 \cdot 10^{-3} = 716,2 \text{ kN} \]

\[ \Delta M = N_{bal} \left( \frac{h}{2} - \frac{\xi_{co} d}{2} \right) = 716,2 \cdot \left( \frac{350}{2} - \frac{0,49 \cdot 314}{2} \right) \cdot 10^{-3} = 70,2 \text{ kNm} \]

\[ M_s = A_{s1} f_{yd} z_s = 603 \cdot 435 \cdot 278 \cdot 10^{-6} = 72,9 \text{ kNm} \]

\[ M_{Rd,max} = M_s + \Delta M = 143,1 \text{ kNm} \]

\[ M_{Ed} = N_{Ed} e = 1620 \cdot 0,0277 = 44,87 \text{ kNm} \]

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The point \((M_{Ed}, N_{Ed})\) is inside the diagram: the cross-section is safe!

Numerically:

\[
M_{Ed}(N_{Ed}) = M_{Rd,max} \frac{N_u - N_{Ed}}{N_u - N_{bal}} = 143,1 \cdot \frac{2333,3 - 1620}{2333,3 - 716,2} = 63,12 kNm > M_{Ed} = 44,87 kNm \quad \text{OK, safe!}
\]

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