



Budapest University of Technology and Economics

Department of Mechanics, Materials and Structures

English courses

Reinforced Concrete Structures

Code: BMEEPSTK601

Lecture no. 11:

REINFORCED CONCRETE SECTIONS SUBJECTED TO AXIAL AND ECCENTRIC COMPRESSION

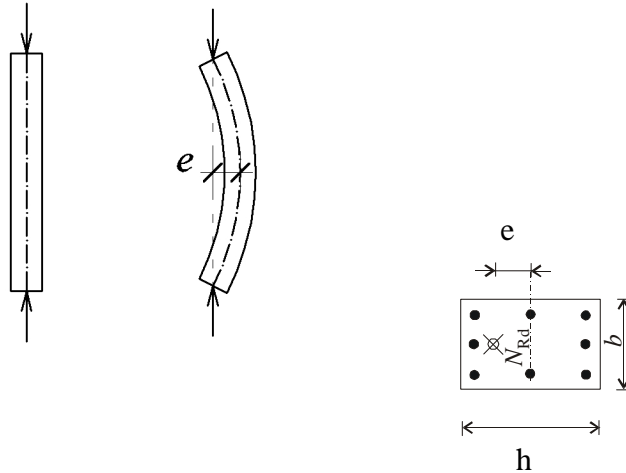
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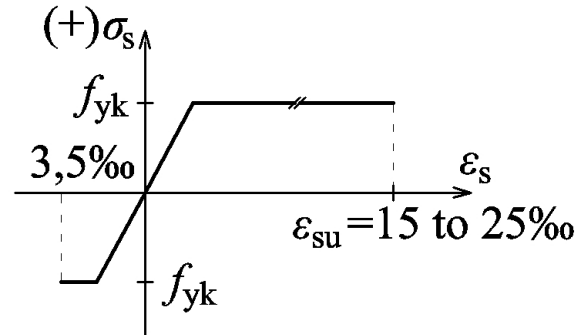
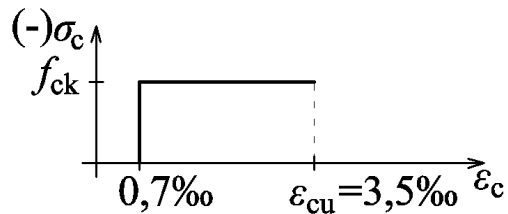
Introduction

The Eurocode 2 does not distinguish axial and eccentric compression, because due to imperfection, the theoretically axial compression force does always have some eccentricity.



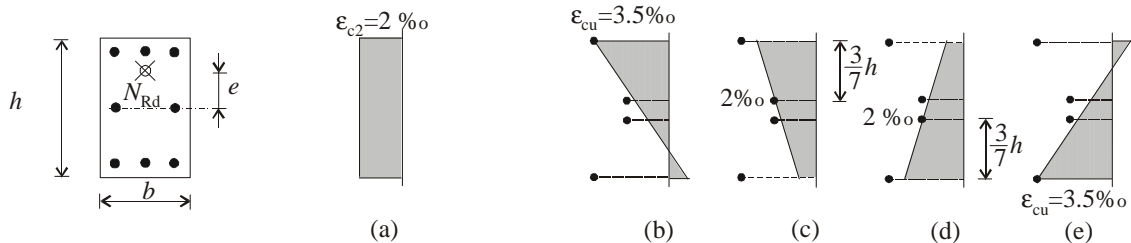
1. Suppositions

- plane sections remain plane
- bound connection between concrete and steel is perfect
- the mechanical behavior of concrete and steel can be modelled by the idealized σ - ϵ relationships indicated below:



2. Failure modes

Failure modes of the rc. section subjected to axial or eccentric compression can be given by the ε -diagrams given below



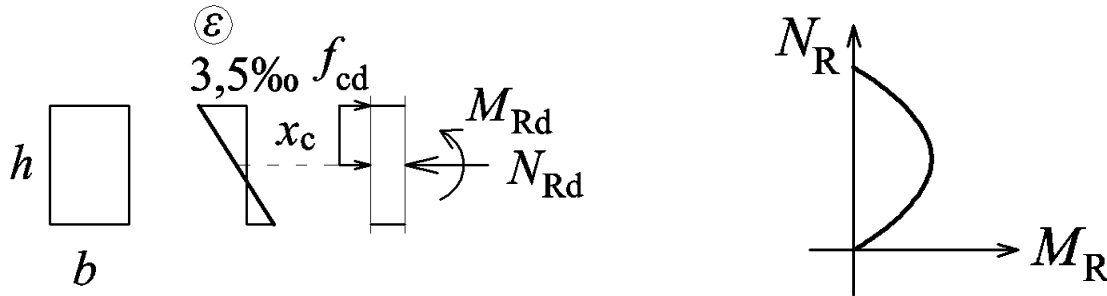
The deformation of the steel at rupture of the concrete extreme fibre is

$$\varepsilon_s \sim \varepsilon_{s1} = \frac{f_{yd}}{E_s}, \text{ that is steel is in the elastic state: } \sigma_s = \frac{560}{\xi_c} - 700 \text{ (N/mm}^2\text{)}$$

For practical design the use of M_R-N_R capacity curves is recommended!

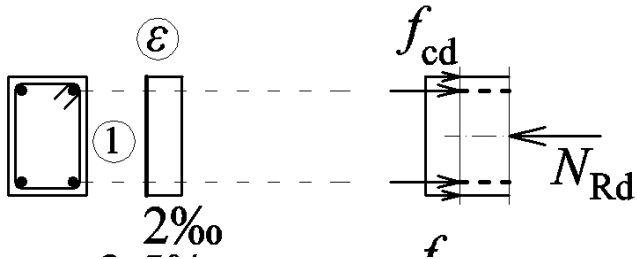
3. The M_R-N_R capacity diagram of concrete sections

For concrete sections the M_R-N_R capacity curve has the character indicated below. Because of zero tensile strength of the concrete, at $N_R=0$, the capacity moment $M_R=0$.



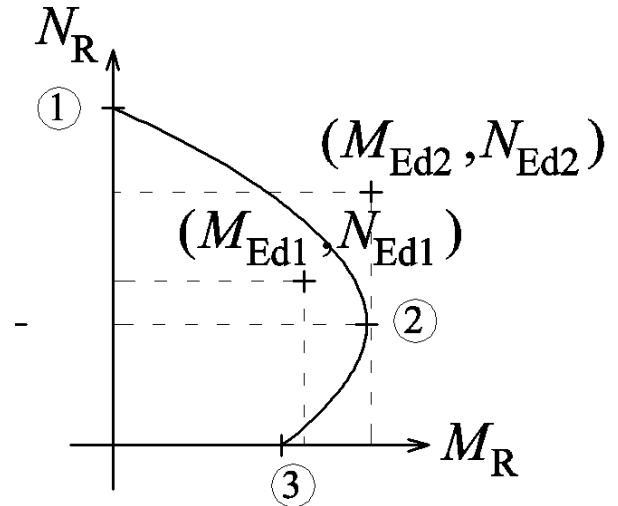
4. The M_R-N_R capacity diagram of reinforced concrete sections

Point 1: axial compression

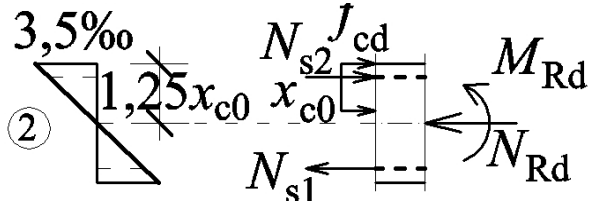


$$N_{Rd} = A_c f_{cd} + A_s f_{yd} = N_u'$$

$$M_{Rd} = 0$$



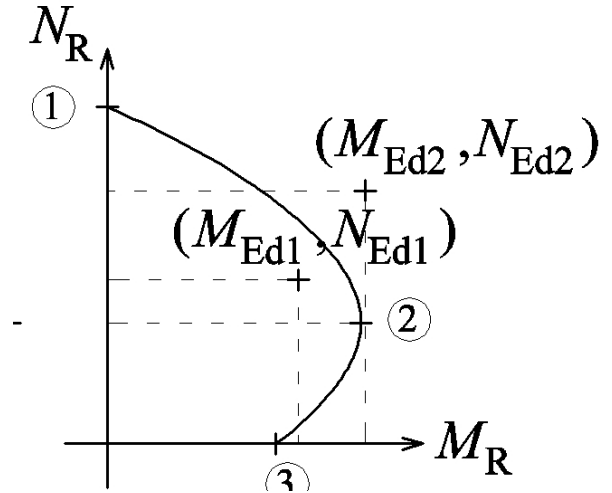
Point 2: eccentric compression ($x_c = x_{c0}$)



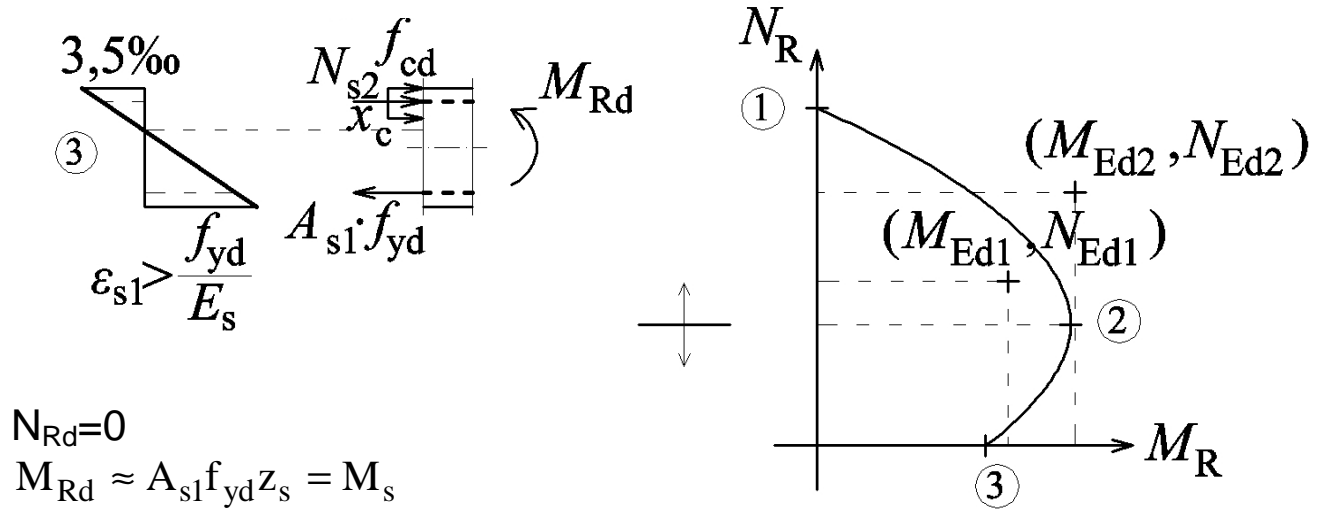
$$N_{Rd} = b x_{c0} f_{cd} = N_{bal}$$

$$M_{Rd} = N_{bal} \left(\frac{h}{2} - \frac{x_{c0}}{2} \right) + A_{s1} f_{yd} z_s =$$

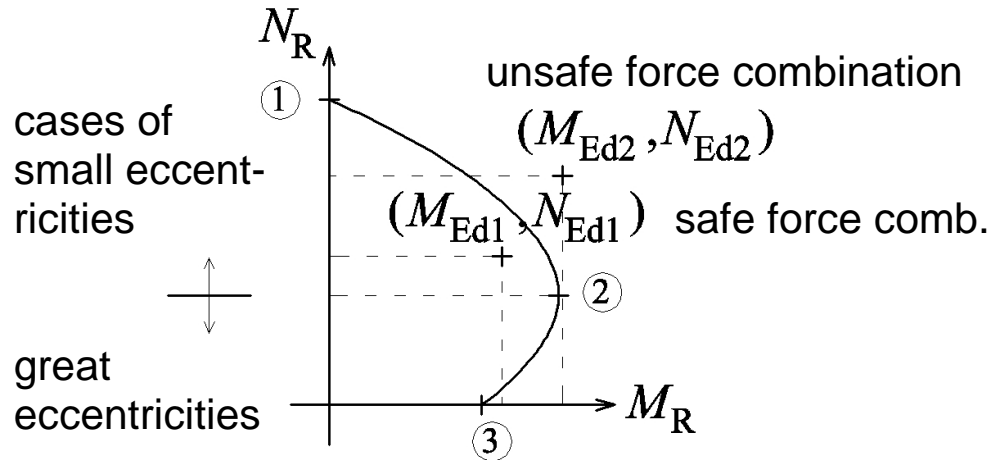
$$= \Delta M_R + M_s = M_{Rd,max}$$



Point 3: flexure



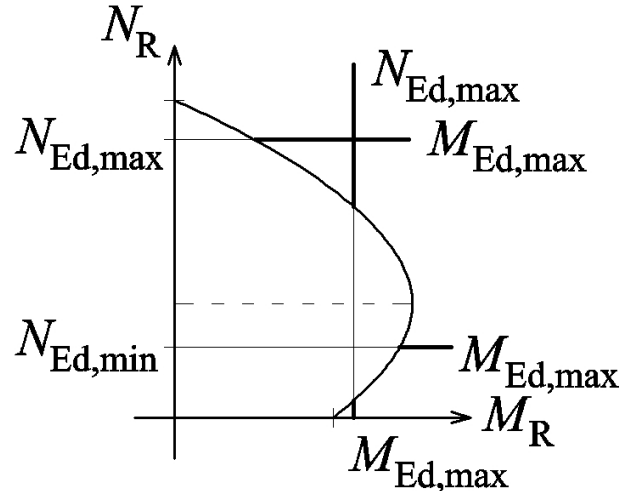
5. Safe and unsafe applied force combinations



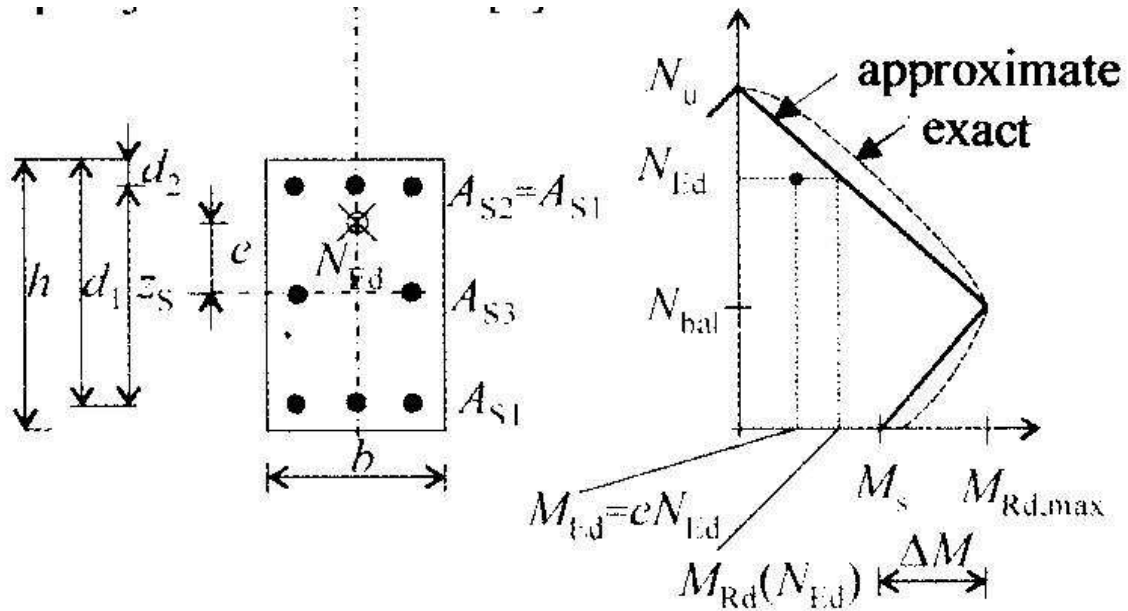
6. Applied force combinations to be investigated

Any of the below force combinations can be dangerous, should be investigated:

1. $(M_{Ed,max}, N_{Ed,min}(M_{Ed,max}))$
2. $(M_{Ed,max}, N_{Ed,max}(M_{Ed,max}))$
3. $(N_{Ed,max}, M_{Ed,max}(N_{Ed,max}))$
4. $(N_{Ed,min}, M_{Ed,max}(N_{Ed,min}))$



7. Linearization of the M_R-N_R capacity diagram

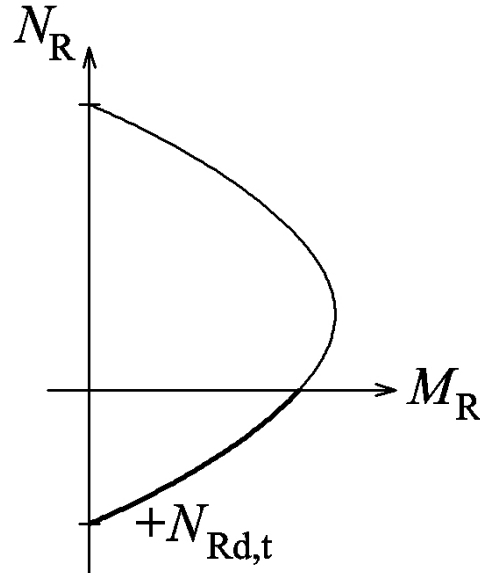


8. Axial and eccentric tension

Part of the capacity diagram below the M_R axis show capacity of the rc section in eccentric and axial tension

For axial tension:

$$N_{Rd,t} = (A_{s1} + A_{s2})f_{yd}$$



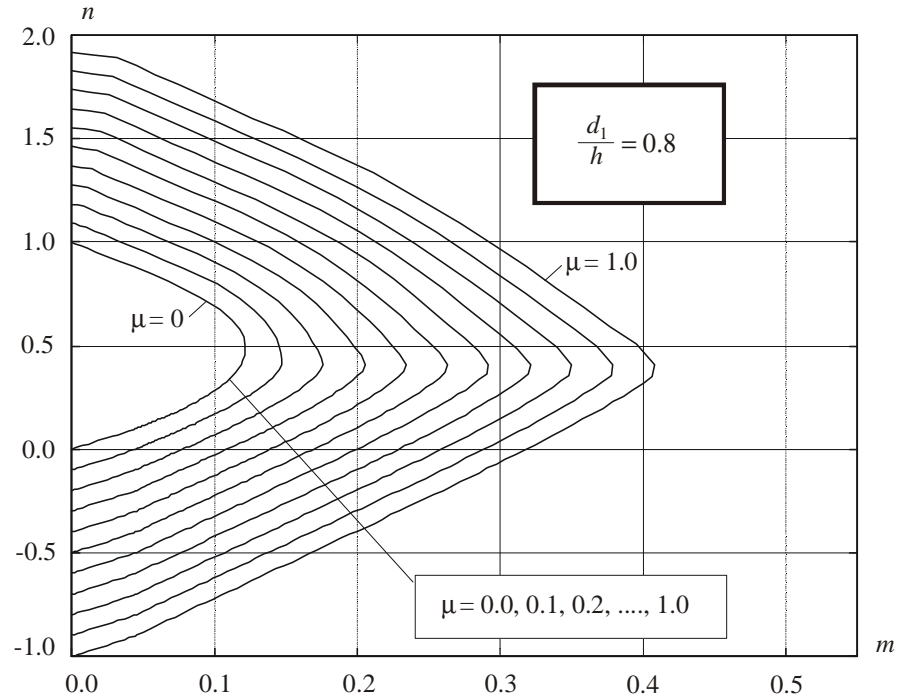
9. Use of a series of dimensionless capacity diagrams

$$\mu = \frac{A_s f_{yd}}{b h f_{cd}}$$

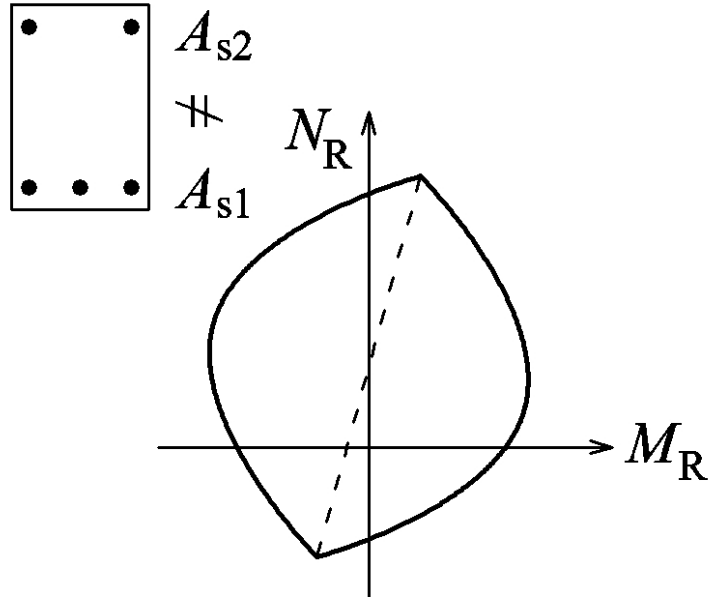
$$m = \frac{M_{Ed}}{b h^2 f_{cd}}$$

$$n = \frac{N_{Ed}}{b h f_{cd}}$$

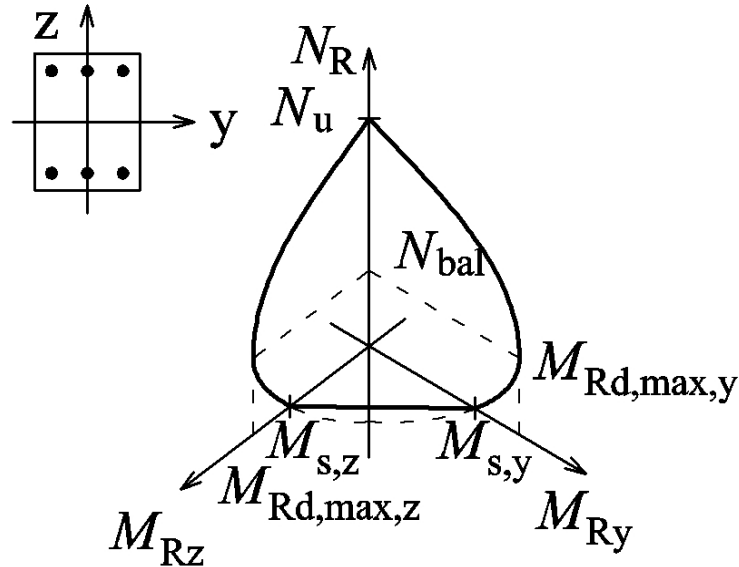
$$A_s = \sum A_{si} = 2A_{s1}$$



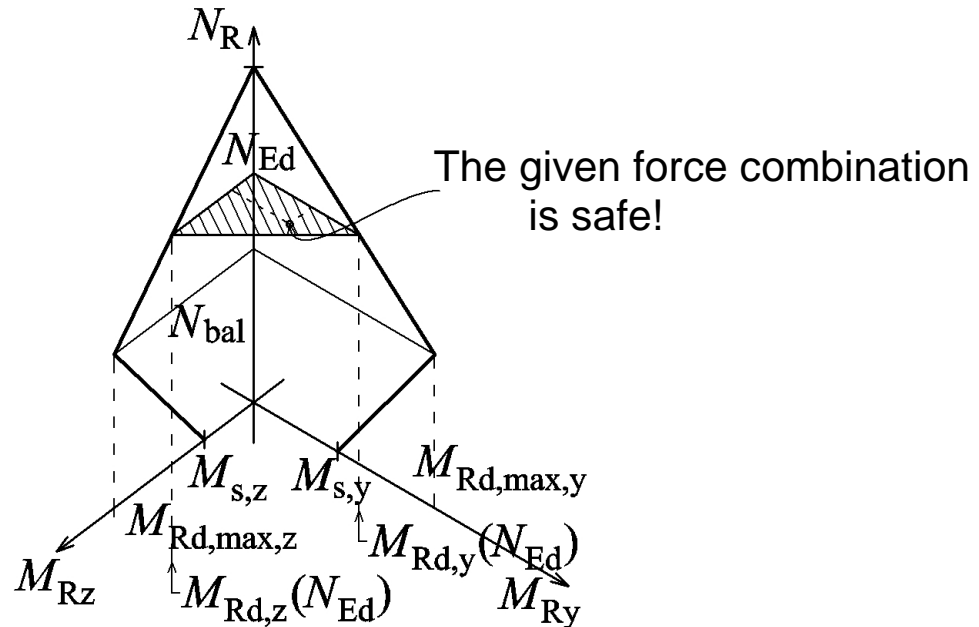
10. M_R - N_R capacity diagram of nonsymmetric reinforced concrete sections

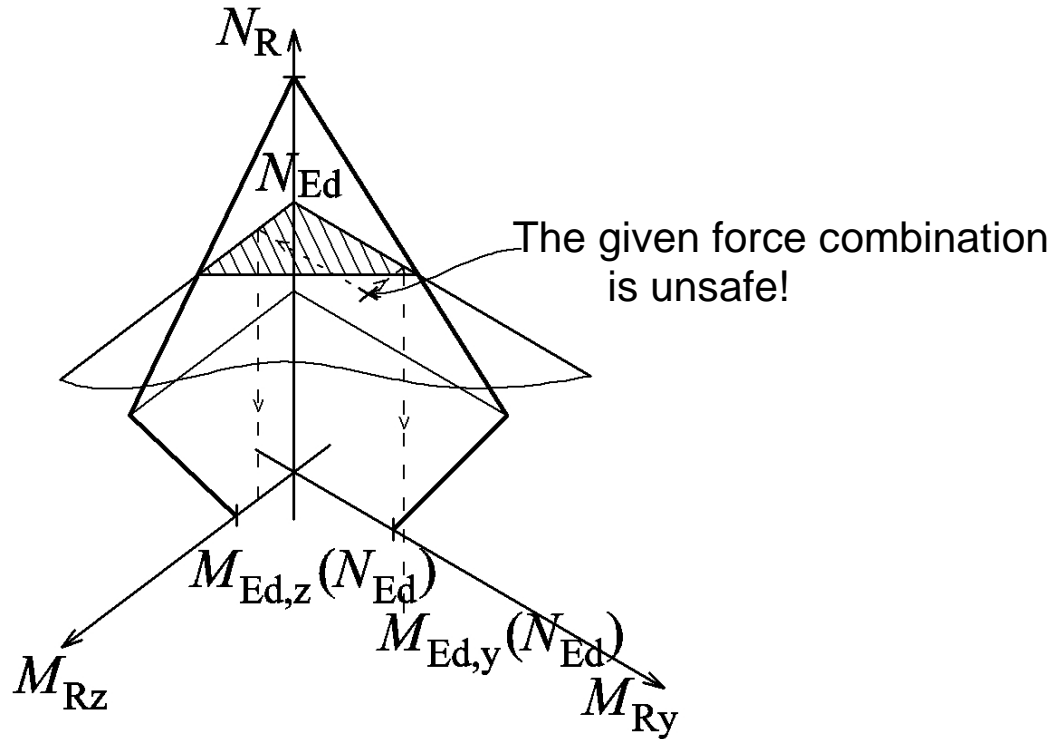


11. The (M_{Rz}, M_{Ry}, N_R) capacity body



12. Check of rc. sections subjected to compression with double eccentricity





13. Numerical example

The column section given below is subjected to an eccentric compression force $N_{Ed} = 1620$ kN, $e = 27,7$ mm.

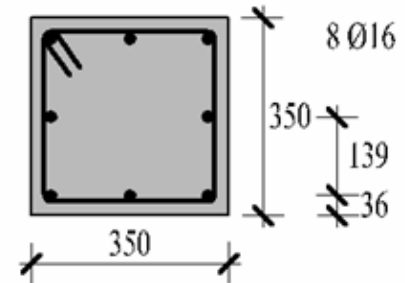
Check the section by determining the simplified (linearized) M_R-N_R capacity diagram of it!

Materials: concrete: C20/25-32/KK steel: B60.50

Concrete cover 20 mm link diameter: $\phi_k = 8$ mm

$$f_{cd} = \frac{20}{1,5} = 13,3 \text{ N/mm}^2 \quad f_{yd} = \frac{500}{1,15} = 435 \text{ N/mm}^2$$

$$\xi_{co} = 0,49$$



Solution:

$$d = 350 - 20 - 8 - 8 = 314 \text{ mm}$$

$$\begin{aligned} N_u' &= bhf_{cd} + A_s \cdot 435 = (350^2 \cdot 13,3 + 1608 \cdot 435) \cdot 10^{-3} = \\ &= 1633 + 700,3 = 2333,3 \text{ kN} \end{aligned}$$

$$N_{bal} = b\xi_{co}df_{cd} = 350 \cdot 0,49 \cdot 314 \cdot 13,3 \cdot 10^{-3} = 716,2 \text{ kN}$$

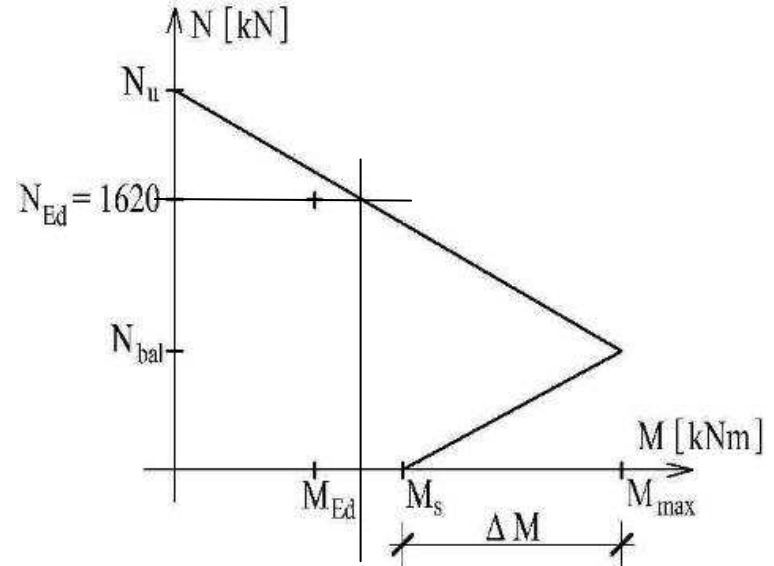
$$\Delta M = N_{bal} \left(\frac{h}{2} - \frac{\xi_{co}d}{2} \right) = 716,2 \cdot \left(\frac{350}{2} - \frac{0,49 \cdot 314}{2} \right) \cdot 10^{-3} = 70,2 \text{ kNm}$$

$$M_s = A_{s1}f_{yd}z_s = 603 \cdot 435 \cdot 278 \cdot 10^{-6} = 72,9 \text{ kNm}$$

$$M_{Rd,max} = M_s + \Delta M = 143,1 \text{ kNm}$$

$$M_{Ed} = N_{Ed}e = 1620 \cdot 0,0277 = 44,87 \text{ kNm}$$

The point (M_{Ed}, N_{Ed}) is inside the diagram: the cross-section is safe!



Numerically:

$$M_{Rd}(N_{Ed}) = M_{Rd,max} \frac{N_u - N_{Ed}}{N_u - N_{bal}} = 143,1 \cdot \frac{2333,3 - 1620}{2333,3 - 716,2} = 63,12 \text{ kNm} >$$

$$> M_{Ed} = 44,87 \text{ kNm} \quad \text{OK, safe!}$$