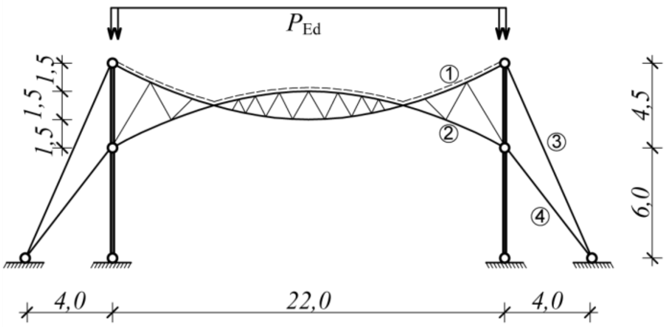
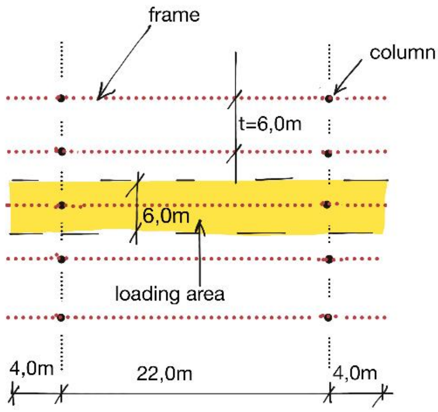
T5 – Calculation of a Cable-Truss Structure

The main load-bearing frame of a hall is a cable-truss structure. The total load on the roof of the hall is P_{Ed}=2,3 kN/m^2, the distance of the frames from each other is t=6,0 m.





The lower and upper cord of the frame are made of cable, the compressed trusses are made of steel rods. The column is (in reality) a continuous, 10,5 m high steel columns, the hinge at 6,0 m height is introduced to simplify the exercise to a statically determinate one. The main internal force of the column is compression hence this simplification is acceptable.

Exercises

1. Calculate the minimum value of prestress applied to element 3. and 4.

2. Calculate the internal forces of the prestressed structure when the external load is applied.

3. Calculate the approximate required cable and column cross sections.

4. Design an economical foundation at the supports of the structure.

Exercise 1. – Calculation of minimum required prestress

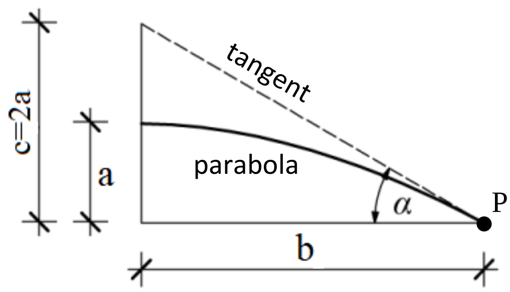
In order to maintain the shape of the truss, we have to ensure that only tension is caused in the upper and lower cord of the truss. The external load causes tension in element 1 and (theoretically) compression in element 2 (it becomes loose). Consequently, all the load is carried by element 1. In order to avoid this, we apply as much prestress in the cables, that the added compression caused by the external load in element 2 will result in a "less tensioned" structure instead of a compressed one.

To calculate the prestressing force, we shall calculate the internal forces of the structure caused by the external load first.

We assume a parabolic shape for the cables. There are only normal forces in the cables, so know that the resultant of the internal forces is axial (or tangential in case of a curved member). First, we calculate the geometry of the tangent. It can be derived, that if the dimensions are a and b of the (half)parabola, then the tangent of the parabola in point P is the hypotenuse of a right triangle with sides c=$2a$ and b (see the figure!).

Then the angle of the tangent line at the hinge (intersection of members 2 and 4): with a = 3 m, b = 11m

$$\operatorname{tg}\alpha = \frac{6{,}0}{11{,}0} \rightarrow \alpha = 28{,}6°$$



The load of one truss from the loading area is: $P_{Ed} = 2{,}3 \times 6{,}0 = 13{,}8$ kN/m.
Calculated on one side the components of the reaction forces at the end of the cables:
$F_{y,ext.load} = P_{Ed} \times (22/2)/2 = 75{,}9$ kN; (we assume that they carry the same forces because of the symmetry)
$F_{x,ext.load} = F_{y,ext.load}/6{,}0 \times 11{,}0 = 139{,}15$ kN *(similar triangles)*

Internal forces in the cables: $N_{1-2,teher} = \sqrt{F_{x,teher}^2 + F_{y,teher}^2} = 158{,}5\,\mathrm{kN}$.

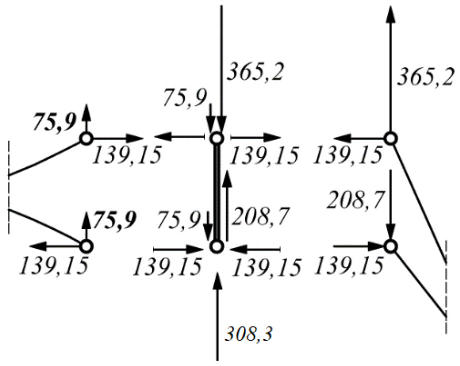
The lower cable (2.) is compressed, the upper one (1.) is tensioned.
The balance of the connecting structures can be determined based on the figure. The vertical components in the binding cables can be calculated based on geometrical considerations:

$N_{3,x,ext.load} = 139{,}15\ \mathrm{kN} \rightarrow N_{3,y,\ ext.load} = N_{3,x,\ ext.load}/4{,}0 \times 10{,}5 = 365{,}2\ \mathrm{kN}$

$N_{4,x,\ ext.load} = 139{,}15\ \mathrm{kN} \rightarrow N_{4,y,\ ext.load} = N_{4,x,\ ext.load}/4{,}0 \times 6{,}0 = 208{,}7\ \mathrm{kN}$

This means that element 3 will be tensioned ($N_{3,ext.load}$ = 390,81 kN), and element 4 will be compressed ($N_{4,ext.load}$ = -250,83 kN).

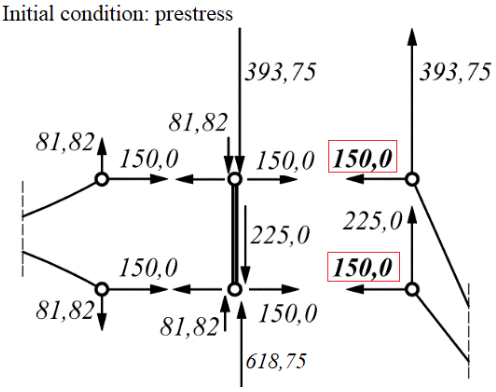
Load: external load



In order to sustain the load bearing capability and to ensure that both cables share the load, as much prestress should be applied to both cables, that element 2 cannot be compressed in reaction to the load.
The minimum value of the prestressing force in element 4 is therefore the calculated $P_{4,min} = N_{4,ext.load}$ = 250,83 kN.

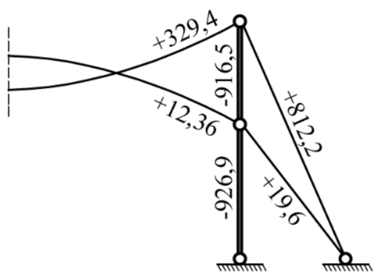
<u>Exercise 2</u> – Calculation of the internal forces
We shall apply as much prestressing force on cable 3 and 4 that their horizontal component of them will be:
$P_{3,x} = P_{4,x}$ = 150,0 kN

Initial condition: prestress



Based on the figure the internal forces caused by the prestressing forces can be determined in all elements. (*No external load is applied yet.*) All elements will be tensioned (*except for the columns*).
Normal forces in the elements:
P_1 = 170,86 kN
P_2 = 170,86 kN
P_3 = 421,35 kN
P_4 = 270,42 kN

Superposition



The reactions caused by the prestressing force and the external load can be added based on superposition *(in case small displacements and elastic behaviour is assumed)*. As a result, the internal force figure can be determined as follows:

<u>Exercise 3</u> – Calculation of the cross sections

Initial data for the calculation of the cable cross sections:

Safety factor: γ_M=2,1.

Fill of the cross section: 50-60%

Tensile strength of the material: f_u = 1550 N/mm^2

The required d diameter can be calculated as: $N_{\mathrm{Rd}} = \dfrac{\frac{d^2}{4}\pi \times 0{,}5 \times f_{\mathrm{u}}}{\gamma_{\mathrm{M}}} \rightarrow d_{\min} = \sqrt{\dfrac{4N_{\mathrm{Ed}}\gamma_{\mathrm{M}}}{0{,}5\pi f_{\mathrm{u}}}}$

The minimal required diameter of the elements:

***d*₁ = 33,7 mm** ($N_{\mathrm{Ed},1}$ = 329,4 kN: load and prestress)

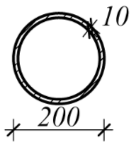
***d*₂ = 24,3 mm** ($N_{\mathrm{Ed},2} = P_2$ = 170,9 kN: *it must bear the prestress force!*)

***d*₃ = 53,0 mm** ($N_{\mathrm{Ed},3}$ = 812,2 kN: poad and prestress)

***d*₄ = 30,5 mm** ($N_{\mathrm{Ed},4} = P_4$ = 270,4 kN: *it must bear the prestress force!*)

Normal force of the column: N_{Ed} = 926,9 kN.

The additional secondary beams provide lateral support at the hinge in 6 m height. Therefore, the buckling length is: $l = l_0$ = 6,0 m.



S235 steel in case of central compression $A_{\mathrm{req}} = N_{\mathrm{Ed}}/f_{\mathrm{y,d}}$ = 926,9×1000/235 = 3944,3 mm^2 cross section is needed.

Because of the slenderness of the structure we'll apply t = 10 mm thick, r = 100 mm diameter pipe-form cross section.

In this case $A_{\mathrm{prov}} = 100^2\pi - 90^2\pi = 5969$ mm^2.

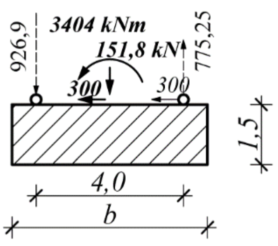
Moment of inertia: $I_{\mathrm{y}} = \dfrac{\pi 100^4}{4} - \dfrac{\pi 90^4}{4} = 27 \times 10^6\,\mathrm{mm}^4$.

Radius of gyration: $i_{\mathrm{y}} = \sqrt{\dfrac{I_{\mathrm{y}}}{A}} = \sqrt{\dfrac{27\times 10^6}{5969}} = 67{,}27\,\mathrm{mm}$.

Slenderness: $\bar{\lambda} = \dfrac{l_0}{i_{\mathrm{y}}\lambda_1} = \dfrac{6000}{67{,}27 \times 93{,}9} = 0{,}95 \rightarrow$ buckling reduction factor: $\boldsymbol{\chi = 0{,}7}$.

Resistance of the chosen cross section: $\boldsymbol{N_{\mathrm{Rd}}} = Af_{\mathrm{y,d}}\chi$ = 5969×235×0,7 = **<u>981,9 kN.</u>** ⟶ **OK!**

<u>Exercise 4</u> – Design of the foundation



The load falling on the foundation will be balanced so that a common foundation will be introduced that counterbalances both tensioned and compressed elements.

The internal part of the foundation is compressed because of the normal force in the column, the external part is tensioned by the cables. The external loads are summed at the top plane in the centre of the foundation:

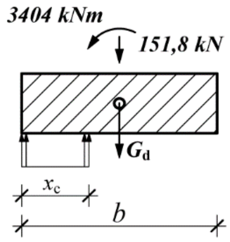
V_{Ed} = 2×150 ==300,0 kN;

N_{Ed} = -926,9 + 775,25 = -151,8 kN

M_{Ed} = 926,9×2,0 + 775,25×2,0 = 3404 kNm

A) We assume equilibrium so that the eccentrical normal force is balanced by the underlying soil (no tensile strength, plastic behaviour). Horizontal (shear) force is balanced by friction and the soil next to the foundation.

We're going to determine how long (b) the foundation has to be so that the eccentrical force can be counterbalanced.
The height of the foundation h = 1,5 m; width w = 2,5 m. Resistance of the soil: σ_{max}= 300,0 kN/m^2



Vertical equilibrium equation:

$x_c w \sigma_{max} = N_{Ed} + G_{Ed} \rightarrow$

$x_c w \sigma_{max} = N_{Ed} + bhw\rho_{vb}\gamma_{inf},$

where ρ_{vb} = 25 kN/m^3; γ_{inf} = 0,9. *(because self-weight is stabilizing!)*

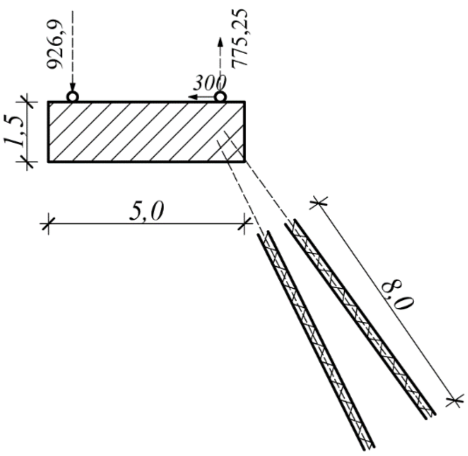
Moment equilibrium equation:

$$M_{Ed} = x_c w \sigma_{határ}\left(\frac{b}{2} - \frac{x_c}{2}\right)$$

Unknown in the equations: x_c and $b \rightarrow$ $b = \dfrac{x_c w \sigma_{határ} - N_{Ed}}{hw\rho_{vb}\gamma_{inf}}$

Solving the equations the real solution is: $\mathbf{x_c}$ **= 1,193 m**.
Consequently, the required width of the foundation is <u>$\mathbf{b_{min}}$ **= 8,8 m**</u>.
The required amount of concrete for this structure: $V = b_{min}hw \approx 33{,}0$ m^3 concrete *(Enough for approx. ~165 m^2 20 cm thick reinforced concrete slab, while one section of the cable structure covers 132 m^2 → not very economical…)*

B) We balance the foundation so that the tensioned side is anchored to soil anchors. 2 pcs. of d = 40 mm diameter, f_u=1550 N/mm^2 strength injected cable anchor will be applied.



The length of the anchors is ~13,0 m, the injected part of this is 8,0 m length. This provides enough grip that the anchors can be fully considered. So:

$$N_{Rd} = 2 \times \frac{\frac{d^2}{4}\pi \times 0{,}5 \times f_u}{\gamma_M} = 2\frac{20^2\pi \times 0{,}5 \times 1550}{2{,}1} = 927{,}5\,\text{kN} >$$

$$\sqrt{775{,}25^2 + 300^2} = 831\text{kN. } \mathbf{MF!}$$

The anchor can bear the tension of the foundation.

The reaction force of the column and the weight of the b = 5,0 m length foundation (G_d = 1,5×5,0×2,5×25×1,35 = 633,0 kN) will be balanced by the soil under the foundation:

$$x_c = \frac{G_d + N_{Ed}}{\sigma_{max} w} = \frac{927 + 633}{300 \times 2{,}5} = 2{,}08\ m$$