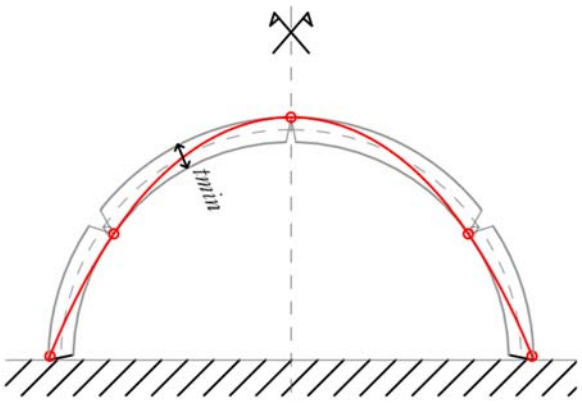
PRACTICAL NR 4 ARCHES

_2019_20_1_handout

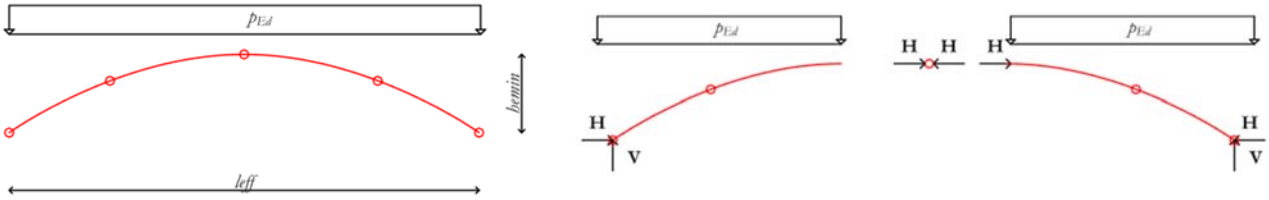
MSc Students

Rotational failure mehanism of the semi circular arch



5 hinge mechanism

Structural model based on the five-hinge mechanism (thrust line in red, hinges indicated*).*



Analysis of existing geometry: data as in the excercise on the flat arch (p_{ed}=25kN/m, l=1,5 m, h=25 cm), however now we would like to know the value of t_{min}! **The support reaction forces are assumed to be the given** (H (28 kN), V (18,75 kN), **same as for the flat arch)!**

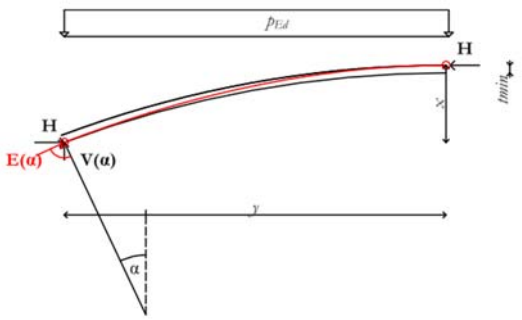
The minimum thickness results from the condition that thrust line cannot exit the boundaries of the arch.

? Determine the critically loaded cross section (most likely to crack)! see failuer mode above.

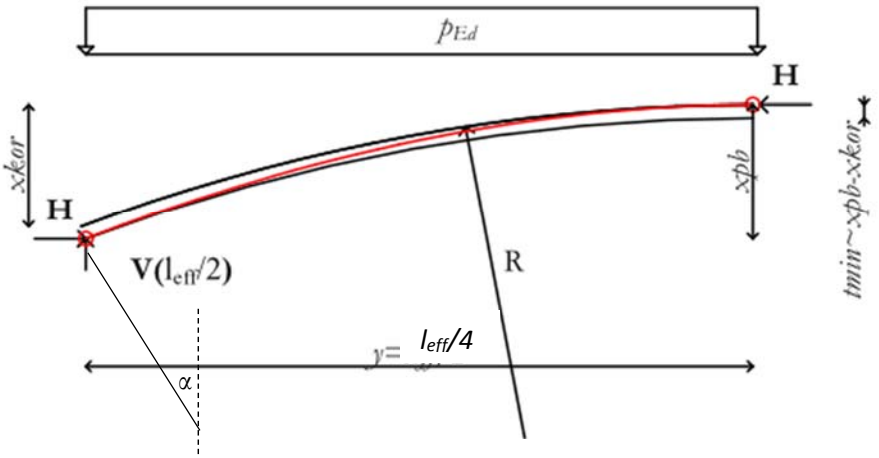
It is at the middle hinge where thrust line reaches the intrados.

Two possible methods to follow:

*(1) * note, that theoretically, minimum thickness analysis assumes the location of the middle hinge unknown. However, the condition that thrust line should reach and be tangent tot he intrados at the location of the middle hinge, suffice (mathematically). Now we use a simplified, less acurate method*



(2) Our estimation is based on the assumption, that the location of the middle hinge is given (at $l_{eff}/4$), then we simply compare the geometry of the (then uniquely determined) resulting parabola (i.e. thrust line) and the circular arc segment, which ultimately results the minimum thickness value.



Steps of calculation:

equation of parabola: $a*(0.75)^2=0.25 \Rightarrow a=0.44$

vertical distance of the middle hinge from the top (note: it lays ont he parabola) :
$x_{pb}(0.375)=0.44*0.375^2=0.062$ m

radius of extrados: $R=(0.75^2+0.25^2)/(2*0.25)=1.25$ m

opening angle from the vertical, at the middle hinge (degree)
$\alpha_{kor}(0.375)=\text{asin}(l_{eff}/4/R)=\text{asin}(1.5/5)=17.45°$

vertical distance of the point corresponding to a radial section through the middle hinge, layng on the extrados:

$x_{kor}(0.375)=R_{ekor}*(1-\cos(\alpha_{kor}(0.375)))=1.25*(1-\cos(17.45))=0.057$ m

rough estimation of minimum thickness $x_{pb} - x_{kor} = t_{min}$ **~0.5 cm**

assuming finite thickness (f_{cd}=1,2 N/mm2): t_{real}=0,5+5= 5,5 ~6 cm thick arch would be necessary)

*(The extra 5 cm is necessary to carry the compressive force, it is based on the formula - 2*x/2, see flat arch)*

A thrust line can always be constructed to any arch (assuming in-plane behaviour) and any given loading based on graphic statics– however, it might or might not be contained within the boundaries of the arch. Note, that there are multiple (infinitely many) solutions, since the general problem (arc hon two fixed supports) is statically indeterminate (see piazza for an interative example). Instead, we assumed the structure to be in the verge of collapse, at its limit state (due tot he formulation of plastic hinges). That resulted a statically determinate problem with geometrical constraints.