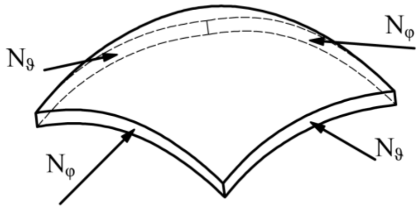
T3/1 – Internal forces of spherical dome loaded by self-weight

Background



The shell structures are often classified according to their force distribution. No shear shells (film), no bending shells (membranes) and bended shells can be distinguished. Only curved shells can carry the load with only membrane forces. Depending on the principal curvatures the internal forces of cylindrical and spherical structures can be calculated. During our calculations we simplify the loads to be symmetrical, perpendicular to the surface and constant in size, moreover, we consider the principal directions to be constant.

Aim of the practical

The calculation of the internal forces of a dome loaded by its self-weight (as the principal load) is presented. The load is varying, and it is not perpendicular to the surface. The stress at each point of the structure can be calculated from the statical equilibrium equations and the geometry. The internal forces at a point in the principal directions depend on the central angle between the point and the apex of the cap. There is compressive stress in the meridian direction, while in hoop direction the sign of the stress changes.

This change in the sign explains the often-observed phenomena in the history of architecture, that spherical domes split into sections at a certain height (e.g. St. Peter's Basilica). But these fissures do not lead to the failure of the structure, they only modify the force distribution. The internal forces in the edge beam also depend on the height of the dome.

The formulas given underneath are valid only for the given loading. In the case of asymmetrical or concentrated loads, the internal force distribution is completely different!

Internal forces of a spherical dome:

- loaded by its self-weight:

$$N_\varphi=\frac{-R\cdot g_d}{1+\cos\varphi}, \qquad N_\vartheta=g_d R\left(\frac{+1}{1+\cos\varphi}-\cos\varphi\right)$$

- loaded by uniformly distributed load:

$$N_\varphi=\frac{-R\cdot p_{Ed}}{2}, \qquad N_\vartheta=\frac{R\cdot p_{Ed}}{2}(1-2\cos^2\varphi)$$

Static equilibrium equations (Barlow's formula):

- in 2 directions:

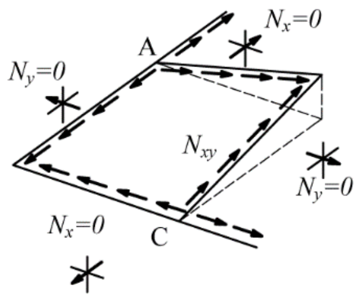
$$\frac{N_\varphi}{R}+\frac{N_\vartheta}{R}=g_d\cos\varphi$$

- in 1 direction:

$$N_{hoop}=N_{\varphi_H}\cdot r$$

T3/2 – Examination of the force distribution in an umbrella like roof (hyperbolic paraboloid)

Background



Apart from the spherical dome, the quadrilateral shell is the most used shell type. It is a ruled surface with straight rulings, which simplifies the construction of the structure.

It is obvious from the structural analysis of the shell, that the normal forces parallel to the rulings are unable to carry the vertical loads. However, the vertical components of the internal shear forces along the edges (N_{xy}) can support the vertical loads. To calculate the internal forces, it would be complicated to use the equations of the principal curvatures, as the radii belonging to the principal curvatures are not available automatically. Nevertheless, due to the simple geometry and simple loading, the forces along the semi-rigid edges can be easily calculated. The internal forces along the principal directions are the same at each point of the structure as a result of the geometry.

Aim of the practical

The internal force distribution of a complex special shell structure is presented. To calculate the internal forces, it is assumed that the linearly distributed load over the surface is approximately equal to the projected load, so the internal shear forces along the edges can be calculated. The emerging shear force along the semi-rigid edges is transferred as a normal force to the supports.

The force distribution of the asymmetrical loading of the quadrilateral structure is also presented. This load distribution results in a much unfavorable force distribution in the supporting structural elements of the shell. Such a load distribution can be the result of asymmetrical snow distribution, which loads only half of the roof.

During the design of the purely compressed semi-rigid edge and the tie bars are presented.