4. Masonry arches and vaults

4.1. MASONRY ARCHES

4.1.1. Introduction
In the beginning, openings were spanned with wooden beams, stone blocks and stone beams which provided only a small span. The development and usage of arches was a cultural and technical milestone in architecture. Arches and vaults became widespread during the Roman Empire. Arches facilitate the spanning of large distances using building materials lacking tensile strength (e.g. brick or stone), by supporting the vertical loads mainly by axial compression. In a beam supported on both ends bending arises unless the shape of the structure follows the shape of the thrust line. Thrust is the outward pressure, and the line of thrust is constructed by the resultants of thrust and weight. Bending can be eliminated by choosing the shape of the structure such way, that the line of thrust coincide with the arch axis.

4.1.2. Terminology

**Abutment**: structural members which support the one end of the arch, vault or strut

**Intrados**: the inner curve/surface of an arch, most of the arches are classified by the curve of the intrados

**Extrados**: the exterior curve/surface of the arch

**Voussoir**: wedge-shaped masonry units

**Crown**: the apex of the extrados, the highest point of the arch/vault. It is common to use a **keystone** at the crown which provides the compressive strain between the voussoirs.

**Springing**: the point at which the arch rises from its support. This is the lowest point of the arch.

**Rise**: the distance between the springing line and the highest point of the intrados. We distinguish the **height of the arch** which is the distance between the springing line and the highest point of the arch axis, but it can be approximated by the rise.

**Span**: the horizontal distance between the abutments

4.1.3. Construction and calculation
To design a bending free structure and to construct the line of thrust, the loads should be known in advance. However, the shape of the structure and the distribution of the forces affect each other. The easiest way to construct the thrust line of structure supported on both ends is to calculate the bending moment diagram of a straight beam having the same span. The bending moment diagram is the **catenary curve** and the **thrust line** is its inverse. The arch shaped according to the thrust line and lacking bending moment is a **funicular arch**. The shape of the bending moment diagram resulting from uniformly distributed load (typical load) is parabola, while the self-weight of the structure leads to a catenary shaped bending moment diagram (\(ch(x)\) f function). The most common shape of an arch in architecture is a **semicircle** which is close to both geometries leading to only small bending moments for both loads.

Requirements of the arch to support the loads:

1. The line of thrust must lie inside the structure.
2. The arising stresses must stay below the strength of the material.
3. Structural stability
The first requirement can be checked with geometric construction (graphically): 1. draw a proportional drawing of the arch (with the appropriate thickness determined from the cross-section), 2. draw the bending moment diagram, 3. stretch the bending moment diagram horizontally/vertically in such way, that the line of thrust must line entirely inside the structure.

Requirements 1. and 2. can also be checked simultaneously: 1. determine the idealized model of the arch, 2. calculate the bending moment, shear force and normal force diagrams, 3. check wether the structure can bear the loads without tensile strength. This method reveals wether the eccentricity of the internal forces is small enough for the thrust line to stay within the structure. However, the critical point is the first step, choosing the appropriate idealized model for the structure. The best fit would be an arc clamped on both ends, because the thickness of the structure is usually uniform along the arch axis (so the resistance and stiffness of the arch and the supports are identical) leading to a 3 times statically indeterminate structure. For the calculation of such structure complicated methods (e.g. force method, finite element method) would be required, therefore it is better to choose a simpler idealized model. We utilize the fact, the largest bending moment arises at the clamps and for increased loading these points yield the first. Since both masonry and stone are prone to plastic deformations the structure would crack but not collapse. The yielded cross sections are called plastic hinges. After the plastic hinges appeared, the idealized model of the structure is a two hinged arc. Examination of its bending moment diagram reveals, that the larges bending moment arises at the two third of the arc and in the midpoint. Suppose, that further increasing the loading would lead to the yielding of the midpoint – the crown – of the arch creating a third plastic hinge. Now the idealized model of the structure is a three hinged arc. The largest bending moment arises at the quarter point of the arc. In summary, before the arch collapses under the increasing loading it can be modelled by an arc clamped on both ends, then a two hinged, and finally a three hinged structure.

We can utilize the fact that we know how the arches collapse. If we choose the axis line of the arch in such way, that it would stay within the structure, but it would lie near the crown and above the theoretical position of the supports we determine the final position of the plastic hinges geometrically. This step is arbitrary, but the stresses in the resulting structure must be checked. The idealized model determined by this technique is a three hinged arc and it is enough the check the structure at the hinges and at the quarter points.

Besides yielding, a typical form of the failure of the structure is the inward movement of the supports. It could be a result of an overstressed tie bar (or tie rod) or the horizontal load as a support reaction force of the neighboring arcs. In case of inward movement, bending moment arises and the largest bending moment occurs in the quarter points (not at the crown as in the previous case) leading to a four hinged mechanism under increasing loading. A four hinged structure is unstable, statically overdeterminate and it leads to collapse. One of its disadvantages is that, the cracks would appear at the upper part of the arch which is visually hidden. To sum it up, movements of the supports should be hindered!
Stability loss is not a typical form of failure in case of arches, because they are usually loaded by walls preventing the buckling of the structure. Barrel vaults are stiffened by transverse arches to prevent buckling.

4.1.4. Typical arches
The total load of a typical arch usually has two components: a uniformly distributed projected part and a uniformly distributed part acting along the arch axis. Although arches are rarely loaded by concentrated forces, they are transferred to the arch by the structural elements above the arch (e.g. spandrel) and transformed into distributed load. The thrust line of the abovementioned distributed loads has a parabolic and catenary shape, respectively. These two curves fit into a semicircular arch leading to the satisfaction of Requirement 1. The same is true for the elliptical arch. It is not necessary to build the whole semicircle (segmental arch), but if the rise is smaller than the radius, horizontal forces arise at the supports.

To reduce the horizontal forces, one solution is to use gothic arch. Its sides consist of arcs of circles, but there is a breaking point at the crown. This solution leads to larger rise and smaller horizontal forces for the same span.
To avoid breaking points, it is possible to construct **multicentered arches**, which consists of several arcs of circles which are tangent at their intersections. The **ogee arch** is a **three-centered arch**, when two large circles are joined with a small one and it was typical for the eastern architecture. Another three-centered arch is the **basket-handle arch**, which is constructed by joining two smaller circles with a large one and it was widespread in the baroque era. However, one disadvantage of the basket-handle arch is that its rise is smaller which leads to larger horizontal support reactions.

Some of the further possible shapes for the intrados are the **cycloid** which is the trace of a point on the circumference of a circle rolling along a straight line) and the **parabola**, but they were rarely built.

A special arch is the **flat arch (Jack arch)**, which has a straight horizontal intrados. The thrust line must lie inside the structure; therefore, it is a shallow parabola and it results in large horizontal support reactions. As a result, the maximal span of a flat arch is 1,3-1,5 m. For most effective force distribution shape of the voussours should be chosen to be perpendicular to the thrust line.

**4.1.5. Support types**

The main problem of the structures following the shape of the thrust line is the horizontal force arising at the supports. The higher the structure, the smaller the horizontal forces, making the gothic arch and the ogee arch the most effective and the flat arch the least effective solutions in terms of the force distribution.

The support must be formed such way, that the thrust line lies inside the structure.
Possible ways to handle the horizontal force:

1. Building **multiple similar arches** (with similar support reactions) next to each other, they can provide the horizontal **support** for each other.
2. **Walls or pillars** can also provide the horizontal support. Requirement for the equilibrium: the resultant of the forces must lie in the structure.
3. If the width of the wall is insufficient, a **buttress** can stabilize the structure.
4. Building **multiple stories** increases the vertical component of the thrust. Consequently, it moves the thrust line closer to the center of the cross section at the support.
5. Adding **extra weight** is widespread in case of gothic cathedrals in the form of pinnacles.
6. If the space for carrying the horizontal force is limited, the best solution is a **tie-bar**. It connects the supports and the horizontal forces arising at sides balance each other. The tie-bar must support the springings, but its location is not restricted, it can be placed above the arch (increasing the bending moment in the wall).
7. Using skew tie-bars, the horizontal forces can be transferred to a **beam** located **above the arch**. In this case, bending moment arises in the beam.

### 4.1.6. Abutments

For the most effective force distribution, the voussoirs should be perpendicular to the thrust line, because the stones/bricks can carry large compression but only small shear forces. The main problem of the springings is that joints of the masonry walls are usually horizontal or vertical while the joints of an arch are skew. These two systems can be joined with a **skewback** or by the **intersection** of the two masonry parts.

If the arch is supported by short wall segments or columns, then it should lie on small cantilever skewbacks to avoid weakening the vertical elements. The skewback can be formed by masonry, an appropriately shaped stone or a steel elements. The skewback angle should be perpendicular to the thrust line.

When the arch is supported by large wall segments, the cantilever elements are not necessary.

The problem with the junction of two different systems is the same in case of the **Jack arch slab**, which is a masonry arch combined with steel I beams. There are two possible ways to handle the joint between the steel beam and the arch: with a cut or cant brick to fit on the I-beam or with a specially shaped masonry unit. The latter method is more desirable in terms of fire safety.
4.2. VAULTS

4.2.1. Introduction

Masonry arches and vaults are close in nature, under uniform load the induced stresses are mainly compressive in both structures. Masonry arches and vaults were developed simultaneously during the Roman Empire. The most significant vault from this era is the Pantheon, which is a composite coffered structure made of concrete panels between brick ribs.

Later, but before reinforced concrete slabs became widespread, vaults and Jack arch slabs were used to cover the basements. Nowadays there are more effective structural solutions to cover large spans and vaults are used only to satisfy special architectural needs (e.g. Eladio Dieste’s architecture).

The stress distribution in vaults and reinforced concrete shells is similar, but the tensile strength of masonry structures is extremely low and it is even zero in some directions. The reason lies in the fact, that although masonry units and stones can bear tension, they are bonded using mortar lacking tensile strength.
There are different bond patterns that is formed by the mortar joints and the masonry units on the face of the wall. A running bond consist of horizontal bed joints and vertical head joints. A running bond wall has no tensile strength perpendicular to the bed joints, but it can bear tension perpendicular to the head joints because mortar have shear strength. Consequently, the bond pattern of vaults should be formed in such way, that the bed joints lie perpendicular to the trajectories of principal compressive stresses and parallel to the tensile stresses.

A special vault is the Catalan vault that is constructed by laying multiple layers of bricks lengthwise. It has tensile strength in all directions, and it is also possible to place steel reinforcement between the brick layers.

4.2.2 Simple vaults
Simple vaults are surfaces of revolution and barrel vaults.

The horizontal bed joints of a dome should be parallel to the circumferential rings, because the meridian stresses are always compressive. As we know, if a dome under distributed load is not shallow enough, tension arises in the lower part of the dome, in the hoop direction. As a result, most of the masonry domes crack along the meridian joints leading to a force redistribution. The resulting meridional segments carry the loads as arches. This is the structural behavior of the dome of the St. Paul’s Cathedral in Rome.

The sail vault can be constructed with semi-rigid support along its pendentives in the form of masonry walls. In case of the edges are supported, the bonding pattern should be built in such way, that the bed joints are diagonal. Subsequently, in case the vault cracks the force distribution changes and the structure behaves as an arch.
The **barrel vault** is basically a wide arch. If the thrust surface lies inside the structure, then the strength of the vault is usually satisfactory to bear uniformly distributed vertical loads. However, kinematic, asymmetric loads or buckling can lead to failure. To avoid buckling, barrel vaults are strengthened by **transverse bracing arches**.

### 4.2.3 Complex vaults

Complex vaults are constructed by the intersection of simple surfaces. Since the tangent direction of two intersecting surfaces are usually different, **ribs** should be placed along the edges to distribute the forces. These ribs provide the support for the **sections**.

![Groin vault](image1.png)  
![Cloister vault](image2.png)

![Dome on pendentives](image3.png)