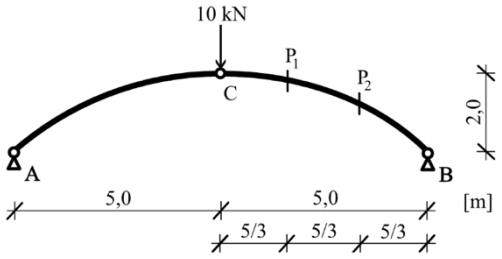
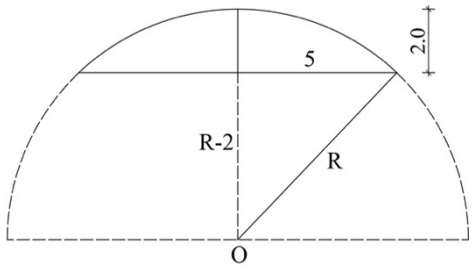
8. Arcs

8.1. A circular arc is given in the figure.
a) Calculate the radius of the circle!
b) Calculate the moment at points P_1, P_2 !
c) Sketch up the moment diagram of the arc!

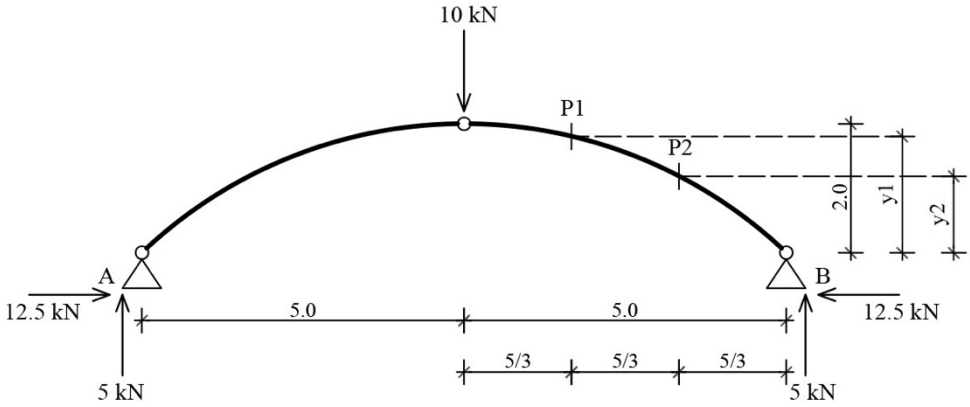


a) The radius of the circle from Pythagoras' theorem:



$5^2 + (R-2)^2 = R^2$
$5^2 + R^2 + 4 - 4R = R^2$
$R = 7.25\ m$

b) To calculate the bending moments, at first we need to determine the heights of points P_1 and P_2:



Support reactions:

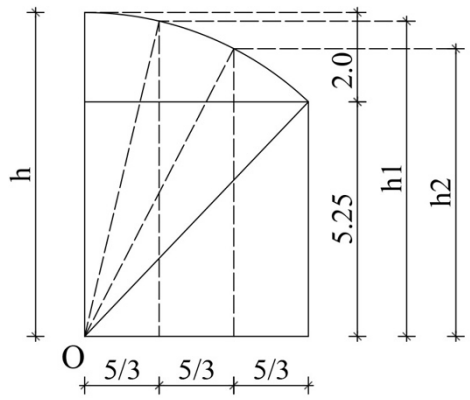
$A_y = B_y = \frac{10}{2} = 5\ kN \uparrow$

$B_x = \frac{5 \cdot 5}{2} = 12{,}5\ kN \leftarrow$

From Pythagoras' theorem:

$\left(\frac{5}{3}\right)^2 + h_1^2 = 7{,}25^2$
$h_1 = 7{,}06\ m$
$y_1 = 7{,}06 - 5{,}25 = 1{,}81\ m$

$\left(\frac{10}{3}\right)^2 + h_2^2 = 7{,}25^2$
$h_2 = 6{,}44\ m$
$y_2 = 6{,}44 - 5{,}25 = 1{,}19\ m$



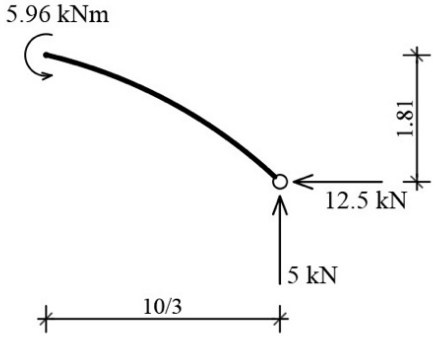
Bending moment:

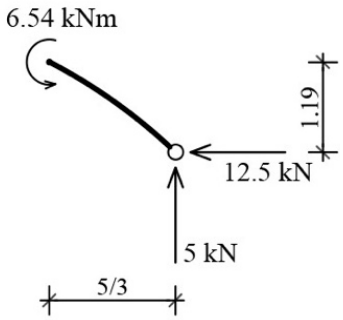
$\sum M_1 = 0$

$M_1 = -5\frac{10}{3} + 12{,}5 \cdot 1{,}81 = 5{,}96\ kNm$

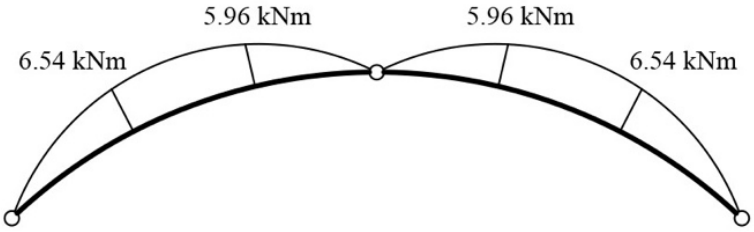
$\sum M_2 = 0$

$M_2 = -5\frac{5}{3} + 12{,}5 \cdot 1{,}19 = 6{,}54\ kNm$

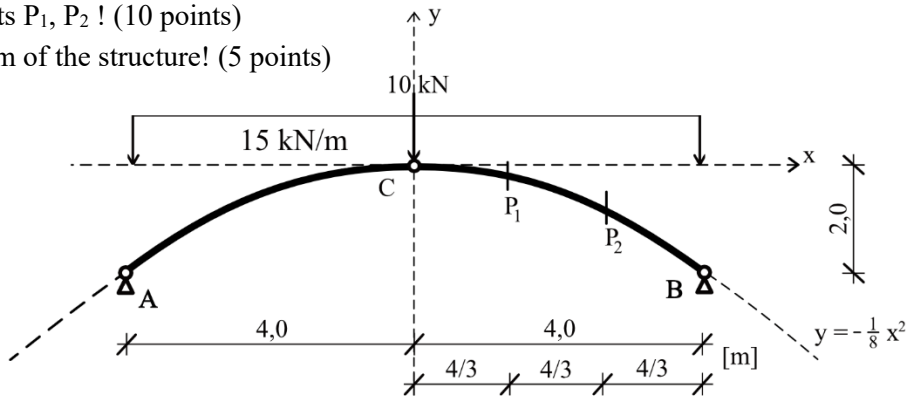




Moment diagram:



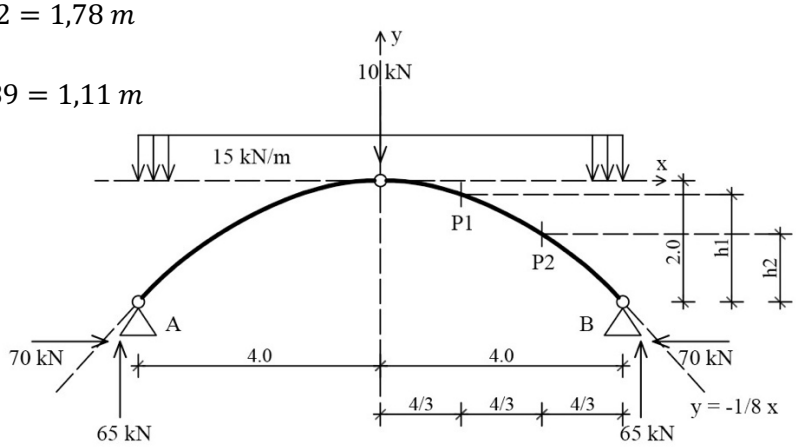
8.2. A parabolic arc is given in the figure. The shape of the parabola is given by the function y(x) in the figure.
a) Calculate the height of points P_1, P_2 ! (5 points)
b) Calculate the moment at points P_1, P_2 ! (10 points)
c) Sketch up the moment diagram of the structure! (5 points)



a) From the quadratic function $y(x)$ the heights can be calculated:

$$y_1 = \frac{1}{8} \cdot \left(\frac{4}{3}\right)^2 = 0{,}22\ m \implies h_1 = 2 - 0{,}22 = 1{,}78\ m$$

$$y_2 = \frac{1}{8} \cdot \left(\frac{8}{3}\right)^2 = 0{,}89\ m \implies h_2 = 2 - 0{,}89 = 1{,}11\ m$$



b) Support reactions:

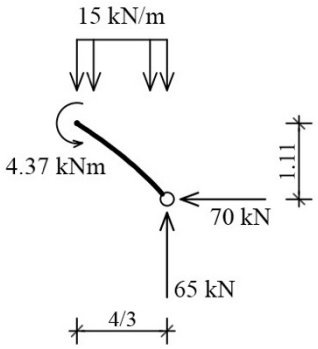
$$B_y = A_y = \frac{15 \cdot 8 + 10}{2} = 65\ kN \uparrow$$

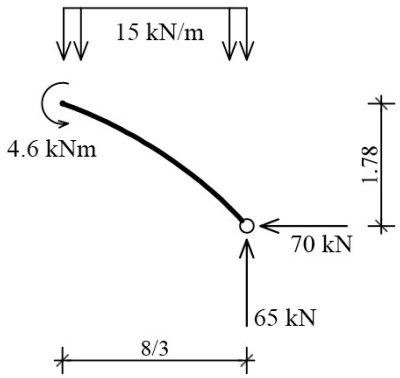
$$B_x = \frac{15 \cdot 4 \cdot 2 - 65 \cdot 4}{2} = 70\ kN \leftarrow$$

Moments:

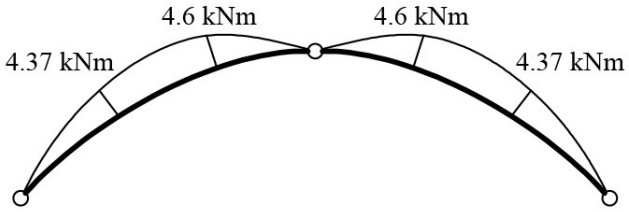
$$M_1 = 15 \cdot \tfrac{8}{3} \cdot \tfrac{4}{3} - 65 \cdot \tfrac{8}{3} + 70 \cdot 1{,}78 = 4{,}6\ kNm$$

$$M_2 = 15 \cdot \tfrac{4}{3} \cdot \tfrac{2}{3} - 65 \cdot \tfrac{4}{3} + 70 \cdot 1{,}11 = 4{,}37\ kNm$$

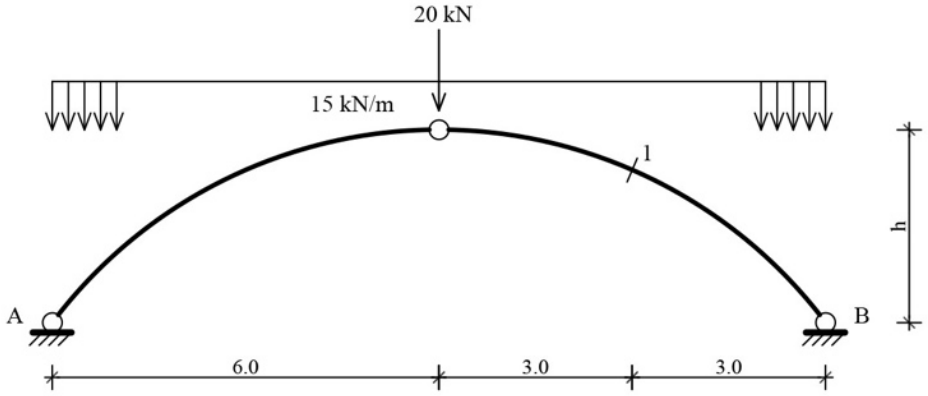




c) Moment diagram



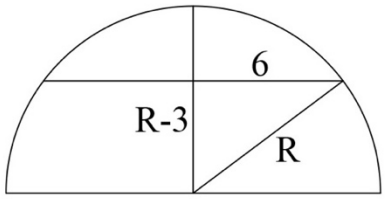
8.3. A circular arc is given in the figure. Calculate the bending moment at point 1, if the height of the structure is h = 3.00 m !



Support reactions:

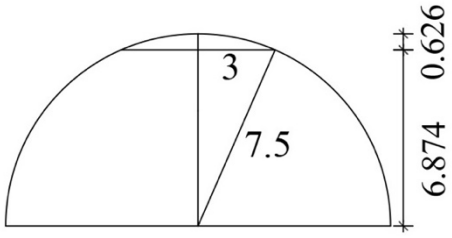
$$A_y = B_y = 6 \cdot 15 + 10 = 100\ kN$$
$$100 \cdot 6 - 16 \cdot 6 \cdot 3 = 3 \cdot A_x \implies A_x = 110\ kN$$

The radius of the circle:



$$6^2 + (R-3)^2 = R^2$$
$$R = 7{,}5\ m$$

The height of point 1:



$$\sqrt{7{,}5^2 - 3^2} = 6{,}874\ m$$
$$7{,}5 - 6{,}874 = 0{,}626\ m$$
$$h_1 = 2{,}374\ m$$

Bending moment at point 1:

$$100 \cdot 3 - 110 \cdot 2{,}374 - 15 \cdot 3 \cdot 1{,}5 = -28{,}64\ kNm$$