Wrinkling behavior of highly stretched thin films

A brief summary of the dissertation submitted to the Budapest University of Technology and Economics in partial fulfilment of the requirements for the degree of Doctor of Philosophy

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1 Introduction

1.1 Motivation

Ultrathin films under compression due to their small bending stiffness prone to a buckling phenomenon, called wrinkling. Generally speaking, being lightweight and having low space requirements, they are easy to transport and can cover large areas cost-efficiently in the form of tensile membrane structures or pneumatic load-bearing structures. These advantageous properties also make them frequently employed in space technology; thus thin films are the main components of solar sails (Figure 1a), inflatable antennas, radars (Talley et al. 2002; Sakamoto & Park 2005) and thin film ballutes (Rohrschneider 2007).

![Figure 1: (a) Models of solar sails (NASA Langley Research Center). (b) Edge waves of a stainless steel coil (Leveltek Processing, LLC). (c) Wrinkling induced by cell motility (Burton & Taylor 1997).](image)

However, uneven surfaces often cause an undesired alteration of the functionality of engineering structures. Accordingly, they are designed to stay wrinkle-free under the design loads. Nevertheless, in most of the cases, wrinkles are impossible to avoid entirely, and the discussion of their source and shape is inevitable. Besides in the engineering practice, wrinkle prevention is also necessary in industrial applications, e.g., in deep-drawing processes (Zhang & Yu 1986), in case of large sheets transported by rolls (Jacques et al. 2007; Yanabe, Nagasawa, & Kaneko 2018), metal coils (Figure 1b) and in biomedical problems such as the healed shape of a surgical wound (Cerda 2005; Hudson & Renshaw 2006). In some cases, wrinkled patterns comprise information that can be resolved by lying on the morphology of thin films. Analyzing the wrinkles on soft elastic substrates induced by cell motility (Harris, Wild, & Stopak 1980; Burton & Taylor 1997; Arocena et al. 2018) (Figure 1c) or the wrinkles on micro-capsules in a flow (Walter, Rehage, & Leonhard 2000) turned out to carry information about the working forces.
Instead of investigating complicated structures under various loads, composed of thin films having complex material properties, most of the research studies related to wrinkling examine simple model problems. In general, a model problem consists of a thin film with a simple geometry subjected to a well-defined load. Although some of these problems are well-understood, the appropriate mechanical models for wrinkling under different conditions as well as the shape and distribution of the emerging wrinkle patterns are still under discussion.

1.2 Model problem

Thin films clamped at two ends while the other two sides are free (Figure 2) wrinkle under axial tensile load. At a critical value of the macroscopic strain $\varepsilon$, the surface buckles, and the sheet becomes decorated with the emerging wrinkle patterns. The wrinkle crests are perpendicular to the direction of the maximal compression.

![Figure 2: The laterally contracted, stretched film under $\varepsilon$ macroscopic strain. Two edges of the rectangular domain are clamped and the other two are free. The dashed lines represent the undeformed rectangular domain in the reference configuration.](image)

Buckling of plates and wrinkling of thin films under compression was a well-known phenomenon, but the wrinkling behavior of stretched sheets was first introduced by Friedl, Rammerstorfer, & Fischer 2000 and gained interest after the publications of Cerda, Ravi-Chandar, & Mahadevan 2002. The mechanical background of the observed phenomenon is still under discussion. Friedl, Rammerstorfer, & Fischer 2000 considered the problem as a superposition of axial stretching and transversal contraction resulting from the Poisson effect. Consequently, the emerging compression was attributed to the restrained transversal displacement at the boundaries. However, Silvestre 2016 revealed, that warping shear also plays an essential role in the wrinkling phenomenon besides the Poisson effect.
Contrasting numerical, analytical and experimental results appeared in the literature regarding the problem. According to Cerda & Mahadevan 2003 the amplitude of the wrinkles increases as the stretch is increased. However, numerical computations with ABAQUS (Nayyar, Ravi-Chandar, & Huang 2011) and physical experiments (Nayyar, Ravi-Chandar, & Huang 2014; Zheng 2008) indicated that the amplitude decreases after the macroscopic strain exceeded a specific value. Furthermore, in the numerical simulations, the amplitudes eventually reached zero at large stretches, but the interpretation of the results was unclear, and the experiments available in literature were unsuitable to verify the phenomenon.

![Figure 3: Wrinkling depending on the aspect ratio and the macroscopic strain according to the eFvK theory, where $\beta = L/(2W)$ is the aspect ratio. For certain aspect ratios wrinkles appear and disappear as the macroscopic strain is increased, but for a fixed length sufficiently narrow or wide films do not wrinkle.](image)

Healey, Li, & Cheng 2013 abandoned the small strain assumptions in the popular Föppl-von Kármán theory (from now on: FvK theory) frequently used to model wrinkling and derived a finite strain extension (from now on: eFvK theory). They carried out a rigorous bifurcation analysis which pointed to the errors of the small strain assumption. Based on the eFvK theory they predicted an isola-center bifurcation in the $\beta - \varepsilon$ plane, where $\beta = L/(2W)$ is the aspect ratio of the undeformed film. In particular, it predicted, that wrinkles disappear as the macroscopic strain is increased and they occur only for a bounded range of aspect ratios. For a fixed thickness, the bifurcation points determine the stability boundary on the $\beta - \varepsilon$ plane.
1.3 Goals

As summarized above, the wrinkling phenomenon is analyzed by several mechanical and material models, different numerical methods leaving open questions that could not be resolved based on the few experiments available in the literature. Moreover, comparability of experimental and theoretical results in the literature is often questionable due to the lack of understanding of the effect of material properties. Most of the works neglect the material nonlinearities and anisotropy, although polymers used in the experiments are often orthotropic as a result of the fabrication process. To be able to apply the findings to complex, real-life structures it is necessary to clarify the applicability of the theoretical models and the effect of material properties.

The main subject of this work is to explore the wrinkling behavior of the model problem in the $\beta - \epsilon$ parameter plane both theoretically and experimentally. In previous experimental and numerical studies phenomena originating from mechanical and material properties were often handled together leaving it unclear what is the primary cause of the observed behavior. We aim to strictly separate material effects and carry out new experiments to address some of the controversial topics. The first and foremost goal of this study is to

- (1) Experimentally verify the predictions of the eFvK theory on the disappearance of wrinkles and the bounded stability region.

Our subsequent investigations are based on the eFvK theory and motivated by experimental observations and the intention to gain a deeper understanding of how wrinkling is affected by different material and geometrical parameters. Furthermore, we extend the model into directions that can be beneficial in the modeling of real-life structures and answer the following questions:

- (2) What is the effect of material orthotropy on wrinkling?
- (3) How inelastic material properties affect the wrinkling behavior?
- (4) What is the effect of intrinsic curvature?

Finally, we turn back to the original problem to investigate a more extended region of the parameter space by letting the thickness arbitrary small. By examining the $\epsilon \to \infty$, $\beta \to \infty$ directions, we raise the following question:

- (5) What are the possible wrinkling configurations in the $\beta - \epsilon$ plane and what is the physical explanation for the observed behavior?
2 A finite deformation linear elastic model

2.1 Theoretical background

The FvK theory assumes linear elasticity, isotropy, and large deflections but small strains. The total potential energy is the sum of the energies related to the bending and the membrane behavior in the model. It takes the assumptions of the Kirchhoff plate theory, the mid-surface represents the plate in two dimensions and its normals stay perpendicular to the surface during the deformation. While the FvK model linearizes the strain tensor in the membrane energy, the eFvK theory (Healey, Li, & Cheng 2013; Li & Healey 2016) keeps the nonlinear terms. The eFvK theory incorporates the Saint Venant-Kirchhoff hyperelastic material model.

The first variation of the energy leads to a fourth-order, nonlinear system of partial differential equations and no closed form of their solution is known. Nevertheless, the shape of the wrinkled pattern can be computed with Newton iteration based on the finite element method accompanied with numerical continuation. Due to the fourth order terms, the standard finite element discretization requires elements with $C^1$ continuity. In contrast, using the Discontinuous Galerkin method (Rivière 2008) it is enough to use elements with $C^0$ continuity. Brenner et al. derived the technique for fourth order problems and the FvK theory (Brenner & Sung 2005; Brenner, Neilan, et al. 2017). To examine the stability of the solutions obtained by Newton iteration we use the smallest eigenvalue of the Jacobian, derived from the second variation of the energy. If the trivial, unwrinkled solution becomes unstable, it indicates a bifurcation point.

2.2 Orthotropic extension of the model

Elastomers are ideal and popular candidates for the experimental investigation of the model problem. Although they are accessible, thin and flexible enough, their fabrication process results in a small orthotropy (Ward 1997). Assuming $x$ and $y$ to be the main material directions, the eFvK model can be extended to orthotropic materials with the introduction of nondimensional orthotropy parameters $r = \frac{Y_x}{Y_y}, q = \frac{S_{xy}}{Y_y}$, where $Y_x, Y_y, S_{xy}$ are the elastic moduli and shear modulus respectively.

We examine the effect of orthotropy in the $\beta – \epsilon$ parameter space. According to the predictions of the eFvK theory, for a fixed thickness, the parameter pairs with a stable wrinkled solution form a closed region in the $\beta – \epsilon$ plane. Healey, Li, & Cheng 2013
showed in a lemma that the negativity of the 2nd Piola-Kirchhoff stress tensor (\(\mathbf{N}\)) is a necessary condition of wrinkling. Therefore, \(\mathbf{N} < 0\) determines the region containing the possibly wrinkled parameter configurations. The effect of orthotropy is compared to the isotropic \((r = 1)\) case. Numerical analysis showed that when the transverse elastic modulus dominates the axial modulus \((r > 1)\), the bifurcation points for a fixed thickness occupy a more extended region and the region of the possible wrinkled parameter configurations is also larger, otherwise \((r < 1)\) they both reduced. In other words, increasing the transverse elastic modulus induce and amplify wrinkling (Fig. 4). The results are summarized in Principal result 1.

![Figure 4: The effect of orthotropy. (a) The grey area represents the possible wrinkled parameter configurations, outside that \(\mathbf{N} > 0\). (b) Points on the sheet where \(\mathbf{N} < 0\) are marked with dark color. (c) The continuous lines represent the stability boundary for different thickness values, while dash-dot lines indicate the border of the possible wrinkled configurations.](image)

### 2.3 Experimental validation of the predictions of the eFvK theory

The experimental examination of the problem requires an ultrathin film being able to sustain large strains without damage. The most popular materials used for such experiments in the literature are silicone rubber (Zheng 2008) and polyethylene (Cerda & Mahadevan 2003; Nayyar, Ravi-Chandar, & Huang 2014). Unfortunately not only
their material behavior is nonlinear, but they also have plastic deformations, which both make it hard to compare the experimental results with numerical computations or theoretical results derived assuming linear elasticity.

![Image](image1)

**Figure 5:** Appearance and disappearance of wrinkles on a polypropylene sheet recorded at different macroscopic strains ($L = 100$ mm, $W = 50$ mm, $h = 28 \mu m$).

Firstly, we carried out experiments on rectangular polypropylene films having $L = 100$ mm length, $W = 50$ mm width and $h = 28 \mu m$ thickness, covered with a 16 $\mu m$ glue layer. According to the factory information, the Poisson’s ratio of the film fluctuates between $\nu = 0.40$ and 0.45. We performed displacement controlled pull-tests using a Zwick Z150 tensile testing machine at the Czakó Adolf Solid Mechanics Laboratory of the BME Department of Mechanics, Materials and Structures. The emerged wrinkled patterns were recorded using side lightning. In agreement with the predictions of the eFvK model, as the macroscopic strain was increased, wrinkles first appeared on the initially flat surface, their amplitude increased then decreased, and finally, they disappeared (Figure 5). This observation qualitatively verified the first prediction of the finite strain model and the error of the small strain model. However, the polypropylene film was not suitable for quantitative comparison due to its large plastic deformations and material nonlinearities.

![Image](image2)

**Figure 6:** Appearance and disappearance of wrinkles on polyurethane sheets ($L = 53$ mm, $W = 25$ mm, $h = 32 \mu m$)

The second set of experiments were completed on rectangular polyurethane sheets having $h = 32 \mu m$ thickness (Figure 6). Analysing the $\sigma - \varepsilon$ (stress-strain) diagram of the film resulted under cyclic loading (Figure 7a) revealed, that it behaves almost
linear elastic after the first loading and unloading. Since different loading speeds resulted in the same $\sigma - \varepsilon$ diagrams, the speed was fixed at 120 mm/min. By examining the wrinkling of prestressed films, it is possible to eliminate material nonlinearities in the model problem. However, the prestress results in a small residual strain and the initially small orthotropy of the film becomes significant. We measured and incorporated $r = 1.8, q = 0.94$ in the numerical simulations.

Figure 7: (a) Applied strain $\varepsilon$ vs. engineering stress $\sigma_{eng}$ (measured force per unit reference area) measured for $W \in \{18, 20, 25, 30, 35, 40\}$ mm, and fixed $L = 50$ mm long sheets. Blue lines denote the trend and standard deviation of stress during the first loading and the unloading (the prestress), and the red line is the reloading. (b) Comparison of computed and measured value of the critical stretch $\varepsilon_{cr2}^r$, where the wrinkles disappeared on prestressed polyurethane films. The dot-dashed line is the border of the possible wrinkled configurations. The grey area is the region of parameter configurations leading to stable wrinkled solutions in the numerical computations.

The length of the examined prestressed rectangles was fixed at $L = 53$ mm and the critical stretch for the second bifurcation point was determined for various width values both experimentally and numerically based on the orthotropic extension of the eFvK model. In agreement with the predictions of the model, wrinkles emerged only for a bounded range of the aspect ratio, rectangles wide or narrow enough exhibited no wrinkling independent of the applied macroscopic strain. Comparison of the experiments and the numerical computations led to the quantitative validation of both predictions of the eFvK model (Figure 7b). The results are summarized in Principal result 2.
3 A pseudoelastic model

Cyclic loading of axially stretched, rectangular polyurethane films revealed an intriguing phenomenon. For some aspect ratios, the sheet remains flat after the first loading, but it wrinkles during the unloading and the subsequent loading cycles (Figure 8).

![Figure 8: Wrinkling of a polyurethane film (W = 35mm, L₀ = 50mm).](image)

Both the unrecoverable deformations and the stress-softening of the applied polyurethane films point to the so-called Mullins effect (Dorfmann & Ogden 2004; Ogden & Roxburgh 1999). We used the same polyurethane sheet as previously to determine the material properties and the wrinkling behavior. The measurements were carried out on L₀ = 50mm long sheets, with an aspect ratio defined by β₀ = L₀ / (2W), at a speed of 120mm/min. Two series of experiments were carried out:

- Traditional displacement controlled pull-tests to obtain a force-displacement diagram for the material (Figure 7a).
- A series of loading-unloading cycles of specimens with different aspect ratios to determine the disappearance of wrinkles during loading εᶜʳ₂ and the appearance of wrinkles during unloading εᶜʳ₃ visually (Figure 8).

We aimed to model the cyclic loading process using a pseudoelastic material model and compare the experimental and numerical results. To accurately model the material upon loading, we incorporated the Mooney-Rivlin material model. We argued that the problem is dominantly uniaxial and suggested a significantly simplified model. In contrary to the classical pseudoelastic model with two dissipation fields, our model is characterized by a single state variable and five material parameters.
Figure 9: Comparison of experimental and numerical results for the disappearance $\epsilon_{cr2}$ and reappearance $\epsilon_{cr3}$ of the wrinkles. Dashed lines denote the examined parameter regions. The grey area represents the parameter configurations leading to stable wrinkled solutions in the numerical computations.

Figure 10: (a) Applied strain $\epsilon$ vs. engineering stress $\sigma_{eng}$. Blue lines and error bars correspond to the measured data, black lines depict the model prediction. (b) Critical macroscopic strain for the disappearance and reappearance of wrinkles. Wrinkling appears only during the unloading for $\beta_o = 0.71$ in both the experiments and the numerical computations.
The material parameters of the model are first fitted to the measured stress-strain diagrams to capture the stress-softening and the residual strain (Figure 10a). We found good agreement between the measurements and computations (Figures 9 and 10). In summary, the model accurately predicts the observed behavior and inelasticity can qualitatively affect the wrinkling of highly stretched thin films. The results are summarized in Principal result 3.

4 Outlook on wrinkling of curved surfaces

To investigate the wrinkling of curved surfaces, it is a reasonable choice to construct and analyze analogies of two-dimensional problems. In the experiments, we compared the shape of unsupported edges of tensile structures and examined the direction of the wrinkles on a model of a complex tensile structure in failure situations (Fig. 11).

![Figure 11: (a)-(b) The shape of an unsupported edge on a flat and a curved surface. Choosing an appropriate initial shape of the edge can prevent the rolling up. (c) Model of a tensile structure in a failure situation. The wrinkles join the highly loaded, disturbed corners of the structure.](image)

There are many extensions of the FvK equations to curved surfaces, yet they are based on the small strain assumption and often restricted to a particular geometry (Donnell 1933; von Kármán & Tsien 1941; Ciarlet & Gratie 2006). We extended the eFvK model to slightly curved surfaces by incorporating the curvature in the membrane part of the energy and mapping the geometry to the plane to examine the effect of curvature on wrinkling.

The surface analogy of the model problem (Figure 2) is the wrinkling of an axially stretched, but transversally curved rectangular surface clamped at two ends (Figure 12a). Numerical computations revealed that for small curvatures wrinkles appear and disappear as the stretch is increased similar to the two-dimensional problem.
However, as the curvature is increased, the amplitude of the wrinkles decreases and above a critical curvature the cylindrical shell does not exhibit wrinkling (Figure 12). Subsequently, the intrinsic curvature of the surface significantly affects the wrinkling behavior.

Motivated by the results, we further extended the eFvK theory to general, curved surfaces incorporating the intrinsic curvature and the contravariant metric tensor in the membrane strain tensor. Our model does not depend on a specific parameterization of the surface, thus non-conventional curved surfaces can be analyzed in the future as well. The results are summarized in Principal result 4.

Figure 12: Disappearance of wrinkles for a stretched open cylinder and the effect of curvature on the critical stretches. a) A cylindrical shell stretched along its curved edges (the other two edges are free). b) Critical stretches, where wrinkles appear/disappear for thickness $h = 0.01$ and $h = 0.02$. In the shaded area, the wrinkled configuration is stable. The emerging patterns for several cases are plotted in the c)-e) subfigures.
5 Investigation of the parameter space

Nayyar, Ravi-Chandar, & Huang 2011; Friedl, Rammerstorfer, & Fischer 2000 examined the wrinkling patterns and the minimum of the transversal stress numerically depending on the macroscopic strain and the aspect ratio. They pointed out that the aspect ratio plays an essential role in the wrinkling behavior. Here we aim to examine a broader range of the parameters and extend the analysis in the \( \varepsilon \to \infty \) and \( \beta \to \infty \) directions. As it was mentioned in Section 2.2, according to a lemma of Healey, the negativity of the 2nd Piola-Kirchhoff stress tensor is necessary for wrinkling. Subsequently, \( \mathbf{N} \) can be used to determine the point-set of the possible wrinkled configurations in the \( \beta - \varepsilon \) plane. We have already examined the border of this region and its dependence on the orthotropy in Figure 4.

![Computation \( N_{22}^{\min} \)](image)

Figure 13: Numerical computation of minima of the transversal stress in the symmetry axis \( (y = 0) \) depending on the stretch and aspect ratio parameters. The Poisson’s ratio is \( \nu = 0.3 \).

We computed the transversal component of \( \mathbf{N} \) denoted by \( N_{22} \) and determined its minimum in the symmetry axis \( (y = 0) \) \( N_{22}^{\min} \) depending on the \( \beta - \varepsilon \) parameter pairs (Figure 13). Although Figure 13 suggests, that wrinkles disappear as \( \varepsilon \to \infty \), the Saint Venant-Kirchhoff material of the eFvK model limits the \( \varepsilon \) strain by \( \varepsilon_{\text{lim}} \). We analytically showed, that \( \mathbf{N} \) is positive definite for \( \varepsilon \to \varepsilon_{\text{lim}} \) assuming Saint Venant-Kirchhoff material. Our results on the higher critical macroscopic strain \( \varepsilon_{\text{cr2}} \) are definitely smaller than the limit mentioned above, but it is worthy to investigate the more realistic Neo-Hookean model. We analytically showed that the 2nd Piola-Kirchhoff tensor is positive definite using the Neo-Hookean model if \( \varepsilon \to \infty \). As a result, independent of the thickness of the film, if wrinkles appear on the surface, there always exists a critical stretch, where they disappear.
Figure 14: Wrinkling of an elongated polyurethane sheet numerically and experimentally ($\beta = 3.5$, $\varepsilon = 0.1$, $\nu = 0.3$, $h = 10\mu m$). There are two local maxima of the wrinkle amplitudes near the clamped boundaries.

Figure 15: a) Disturbed zones forming near the boundaries on an elongated sheet. b) Illustration of the overlap and superposition of the disturbed zones.

Figure 13 also hints, that $N_{22}^{\text{min}}$ is independent of the aspect ratio above some $\beta_b$ value. Motivated by the numerical computation of the $N_{22}^{\text{min}}$ diagram and the observations for elongated sheets (Figure 14), we introduce the concept of disturbed zones forming near the boundaries (Figure 15a). If $\beta > \beta_b$ the zones are separated, otherwise in case of overlapping, their superposition is assumed (Figure 15b).

Figure 16: Minima of the transversal stress determined using the superposition of the numerical computation of $\beta = 5$. 

Figure 16: Minima of the transversal stress determined using the superposition of the numerical computation of $\beta = 5$.
These assumptions are in agreement with Saint-Venant’s principle (Sternberg 1954), meaning that far from the boundaries, our model problem can be considered as simple uniaxial stretching. We used the numerical result of an elongated sheet $\beta = 5$, to produce the $N^{\text{min}}_{22}$ diagram by shifting the $N_{22}$ function and computing the superposition (Figure 16). The resulting diagram is in good agreement with Figure 13.

The concept of the disturbed zones gives a physical explanation of the experimental and numerical observations. In particular, it explains the bounded region of wrinkling parameter pairs in the $\beta - \varepsilon$ plane for a fixed thickness. Furthermore, it is supposed that if a sheet wrinkles for a fixed $\varepsilon$ at $\beta > \beta_b$, the wrinkles amplitude is constant and they do not disappear as $\beta \to \infty$. It is important to emphasize, that although the problem is nonlinear, linear superposition applies to the stresses. The results are summarized in Principal result 5.

6 Principal results

PRINCIPAL RESULT 1.

(Sipos and Fehér 2016)

Geometrically exact models of finite-deformation nonlinear elasticity are needed to understand the mechanics of highly stretched thin films. The classical Föppl-von Kármán plate theory (FvK) has been extended to the finite membrane strain regime recently. The model, called extended Föppl-von Kármán (eFvK) theory, applies some hyperelastic constitutive relation for the in-plane behavior.

1.1. The extended Föppl-von Kármán model with the Saint Venant-Kirchhoff material assumes isotropic material. Motivated by experimental results, I further developed the eFvK theory to accommodate orthotropic materials, at which the main material directions are parallel to the $x$ and $y$ directions of the reference configuration. In the model I used two, nondimensional parameters, $r$ and $q$ to describe orthotropy.

1.2. I carried out numerical computations to investigate the effect of geometric and material parameters on wrinkling. I found, that at a fixed thickness $h$ the aspect ratio of the domain $\beta$, the macroscopic strain $\varepsilon$ and the ratio of the modulii of elasticity $r$ strongly influence the wrinkling behavior. Compared to the isotropic case the stability boundary, that separates wrinkled and flat configuration in the $\beta - \varepsilon$ plane, significantly extends as long as the transverse elastic modulus dominates the axial modulus ($r > 1$). For ($r < 1$) the stability boundary shrinks.
PRINCIPAL RESULT 2.
(Fehér and Sipos 2014, Sipos and Fehér 2016)

To validate the theoretical predictions of the eFvK model, I carried out experiments on previously prestressed, clamped, rectangular, orthotropic, 32 µm thick polyurethane films.

2.1. I experimentally verified the first prediction of the eFvK model. It predicted, that if wrinkles appear on the initially flat surface as the macroscopic stretch is increased, then there is a maximal amplitude for the wrinkles. Further stretch leads to decrease in the amplitude and at a second bifurcation point the wrinkles eventually disappear: The sheet becomes flat again. The experiments clearly demonstrated the disappearance of wrinkles, and as polyurethane is dominantly elastic, this validated the prediction of the model. Using the orthotropic model I computed the location of the bifurcation points in the parameter space and found good quantitative agreement with the experimental data.

2.2. I experimentally verified the second prediction of the eFvK model. It predicted, that wrinkling should appear only for a bounded interval of aspect ratios. In agreement with the prediction, the experiments clearly demonstrated that for fixed length and thickness sufficiently narrow or wide sheets do no exhibit wrinkling. The computed and measured stability boundaries are in good agreement.

PRINCIPAL RESULT 3.
(Fehér, Healey and Sipos 2018)

The obtained experimental data clearly showed, that inelastic material properties significantly affect wrinkling. In specific, stress softening, emerging residual strain and significant change in the orthotropy parameters were recorded.

3.1. In the experiments with polyurethane sheets I demonstrated, that for specific aspect ratios no wrinkling appears during the first loading of the virgin sheet, but wrinkles emerged during the unloading and the subsequent cyclic loading. The measured data clearly hints that the Mullins effect causes the observed phenomenon.

3.2. I introduced a pseudoelastic extension of the eFvK model to take the Mullins effect into account based on the Mooney-Rivlin material model. This model describes the whole loading program of the experiments. The model uses a single
state variable field (damage field) and it has only 5 material parameters. Yet, it predicts stress-softening, residual strain and emerging orthotropy, in accordance with the measured data. The parameter values at wrinkles appear/disappear, are in fair agreement.

**PRINCIPAL RESULT 4.**
*(Fehér and Sipos 2017; Fehér, Hegyi et. al. 2011)*

The eFvK model assuming Saint Venant-Kirchhoff material model was used to investigate the effect of curvature.

4.1. I extended the eFvK model to moderately curved surfaces to investigate it’s effect on wrinkling. The model incorporates the intrinsic curvature of the surface in the membrane part of the potential energy.

4.2. I demonstrated numerically, that intrinsic curvature of the reference domain reduces the amplitude of wrinkles, and over a critical value of the curvature the clamped, stretched cylindrical surface does not wrinkle at all.

4.3. I extended the eFvK theory to general curved surfaces. The model incorporates finite strains, the intrinsic curvature of the surface and the contravariant metric tensor.

**PRINCIPAL RESULT 5.**
*(Sipos and Fehér 2016)*

Although the model problem is dominantly tensiled, wrinkling is caused by (slight) in-plane compression. To understand some of the observations, the 2nd Piola-Kirchhoff stress tensor $\mathbf{N}$ in the eFvK model was examined analytically and numerically.

5.1. I showed analytically, that increasing stretch makes the wrinkles disappear independently of the aspect ratio or the thickness of the film assuming Saint Venant-Kirchhoff material, because if $\varepsilon$ is at its limit, the 2nd Piola-Kirchhoff stress tensor is positive definite. I further showed, that for a Neo-Hookean material if $\varepsilon \to \infty$, then the 2nd Piola-Kirchhoff stress tensor is positive definite.

5.2. I introduced the concept of the disturbed zones of the stress state forming near the clamped boundaries. It is assumed, that the effect of the boundaries for
a fixed width depends only on the applied stretch. In case of overlap of the disturbed zones, their superposition is assumed. I showed, that this concept has a negligible error and it provides a physical explanation for the numerical and experimental results.
Publications related to the principal results


Other publications


References


